When Inflation is More than a Fiscal Phenomenon: Brazil and the URV

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Abstract

The traditional view holds that a credible fiscal reform eliminating the government's reliance on seigniorage is necessary and sufficient to end hyperinflation (Sargent, 1982). Yet stabilizations typically need an additional coordination device to succeed, such as an exchange rate peg. Brazil's 1994 Real Plan offers a striking example. Inflation continued to rise after the fiscal reform and collapsed only after the introduction of the URV, a coordination device. This paper explains why stabilization may fail even when it is fiscally feasible and how a coordination device solves the problem. To achieve that, I build an analytical flexible-price model in which incomplete common knowledge and strategic complementarities can prevent firms from coordinating during a monetary regime change. A coordination device solves this problem by allowing firms to set their real prices without having to forecast other firms' nominal prices. Using a quantitative menu-cost model, I show that coordination devices substantially reduce the output cost of stabilization, with the URV generating smaller losses than an exchange rate peg when a fiscal reform is already in place.

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1 Introduction

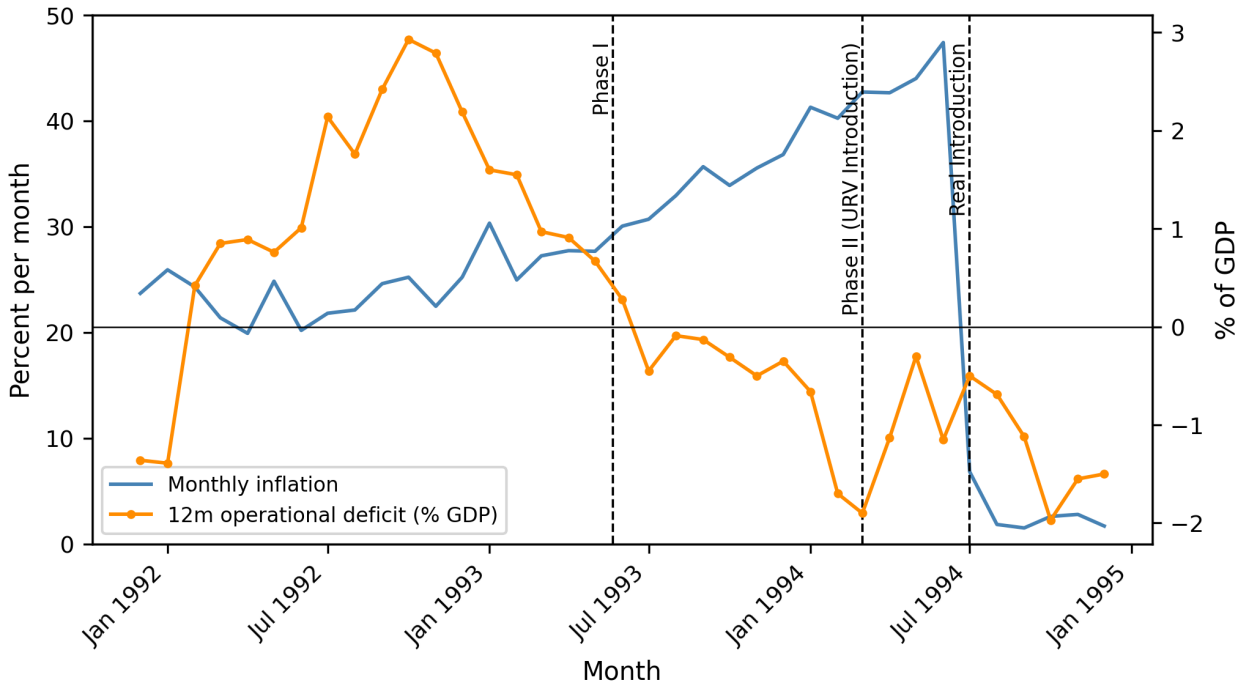
The traditional prescription for ending hyperinflation is straightforward: the government must adopt a credible fiscal reform that eliminates its reliance on seigniorage. Once the need for monetary finance disappears, the nominal money growth rate falls abruptly and brings hyperinflation to an end. Rational expectations allow inflation expectations and prices to adjust immediately, making stabilization fast and costless. On this view, a fiscal reform is a sufficient condition to end a hyperinflation (Sargent, 1982). Yet successful stabilization programs have typically combined fiscal reforms with an additional coordination device. Germany in 1923 and Bolivia in 1985 pegged their exchange rates, Argentina in 1991 and Serbia in 1994 adopted currency boards, and Israel in 1985 used a social pact to restrain nominal wage and price growth. The recurring use of these additional devices is difficult to reconcile with the view that fiscal reform is sufficient to end hyperinflation.

Brazil's Real Plan is a stark example of why the traditional prescription is incomplete. The Real Plan was deliberately divided into two phases: a fiscal reform, completed by March 1994, followed by a purely monetary reform. The monetary reform consisted of the introduction of the URV, a stable unit of account. Figure 1 illustrates this sequencing. By March 1994, the main fiscal measures were in place, and the 12-month operating balance had reached a surplus of roughly 2 percent of GDP.[1] Monthly inflation nevertheless rose from about 30 percent at the start of the reforms to nearly 50 percent. Hyperinflation collapsed only when the monetary reform culminated in the introduction of the Real in July, despite no significant fiscal change or announcement that month. Why did inflation fail to decline right after the fiscal adjustment was in place? Why did it only collapse in July?

This paper argues that the missing ingredient is belief coordination during the transition to low inflation. For disinflation to be costless, all firms have to stop raising prices at the exact same time that the monetary authority announces the new regime. This is particularly difficult during hyperinflation for two reasons. First, large fluctuations in nominal prices and money make it difficult to distinguish a regime change from temporary fluctuations. Second, firms have little incentive to be the first to stop raising their prices. A firm that stops increasing its price while others continue risks a rapid decline in its relative price. As a result, firms have an incentive to keep raising prices to protect themselves against the possibility that others continue pricing for the high inflation. This means that even when

[1]The operating deficit is defined as the primary deficit plus the real component of interest payments on public debt, excluding the part of nominal interest payments that compensates creditors for inflation. It is commonly used in high-inflation episodes because high inflation expectations are embedded in the nominal interest rates, making the nominal deficit extremely large even when the underlying fiscal imbalance is smaller.

Figure 1: 12-Month Operating Balance and Monthly Inflation



Notes: The blue line reports monthly IPCA inflation (left axis), and the orange line reports the 12-month operating balance of the consolidated public sector as a share of GDP (right axis); negative values denote a surplus and positive values a deficit. The dashed lines mark the beginning of Phase I, the beginning of Phase II, and the introduction of the Real. Phase I (May 1993–March 1994) was the fiscal consolidation stage of the Real Plan. Phase II (March–July 1994) began with the introduction of the URV, a stable unit of account in which prices, wages, and contracts were quoted while payments continued in Cruzeiro Real. On July 1, 1994, the URV was converted one-for-one into the Real, Brazil's new currency. Sources: IPCA, IBGE; operating balance, Banco Central do Brasil (BCB).

fiscal reform makes stabilization feasible, firms may remain uncertain about whether the monetary authority will follow through and whether other firms believe it will.

To solve this problem, a monetary authority can make use of a coordination device that provides firms with a common conversion factor from real prices into nominal prices. With this device, firms choose real prices and let the conversion factor give the nominal price. Changes in the price level are absorbed by the conversion factor rather than by firms' real prices. Because real prices are constant when set using the device, firms no longer need to forecast the inflation regime or how other firms will respond to it when setting prices. Incomplete common knowledge no longer distorts real prices or output. As a result, disinflation no longer generates a recession, making the end of hyperinflation costless. To my knowledge, this is the first paper to formalize this.

Most countries use an exchange rate peg as the coordination device. I argue that the Brazilian URV is better suited. The URV was a semi-currency introduced on March 1,

1994, as the Real Plan moved from fiscal to monetary reform. The role of the URV was to be a stable unit of account, in which prices, wages, and contracts would be denominated. Payments continued to be settled in the inflationary currency, the Cruzeiro Real. The conversion rate from URV to Cruzeiro Real was updated daily according to the inflation rate. A firm could keep its URV price fixed while its nominal price adjusted automatically, allowing it to choose a real price without forecasting the inflation rate. On July 1, one URV was converted into one Real, turning the stable unit of account into the new means of payment, ending hyperinflation.[2]

I formalize this argument in an analytical flexible-price model with strategic complementarity and imperfect common knowledge, following Woodford (2001). To isolate the effects of the monetary reform, I assume that the fiscal conditions required for disinflation are satisfied, so fiscal considerations do not constrain the monetary authority's choice. The monetary authority chooses between a high and a low money growth rate, and each firm observes a noisy private signal about that choice. Since the firm's optimal price depends on the aggregate price level, its decision is a function of its own posterior and the full hierarchy of higher-order beliefs. The private-signal structure is related to global-games models of coordination under strategic complementarity (Morris and Shin, 1998). Flexible prices are particularly useful for studying hyperinflation because firms adjust prices very frequently in such episodes (Gagnon, 2009; Alvarez et al., 2019).

The analytical model delivers three results. First, without a coordination device, imperfect common knowledge makes changes in the money growth rate regime have real effects, even though firms can adjust prices freely. Because of noisy signals, if the monetary authority chooses the low money growth rate, firms continue to assign positive probability to the high money growth rate regime and expect other firms to do the same. As a result, inflation falls by less than the reduction in the money growth rate, and output shrinks. If instead the monetary authority chooses the high money growth rate, the opposite happens. Inflation rises by less than the increase in the money growth rate, and output increases.

Second, incomplete common knowledge changes the monetary authority's incentive to stabilize. Disinflation has no output cost under common knowledge, and every monetary authority type chooses the low money growth rate. With imperfect common knowledge, sufficiently output-sensitive types maintain the high money growth rate to avoid the recession. In other words, a fiscal reform alone is no longer sufficient to end hyperinflation.

Third, using the URV as a coordination device removes this cost. Its state-contingent conversion factor absorbs changes in the money growth rate before they enter firms' real-price decisions. The same private signals and higher-order beliefs can remain without affecting

[2]Brazil was not the only country to have an indexed currency. Shiller (1998) surveys the topic.

those decisions. Output is at its natural level under both regimes. A high money growth rate no longer produces an expansion, and a low money growth rate no longer causes a recession. Every monetary-authority type chooses stabilization, making disinflation costless and incentive compatible.

After establishing these analytical results, I build a quantitative menu-cost model based on Golosov and Lucas (2007) and Alvarez et al. (2019) to evaluate the output cost of a disinflation. In the model, the government chooses a distortionary labor tax path constrained to bring debt to a certain level in the long run. The monetary authority reduces the money growth rate and chooses among three different instruments: not having a coordination device (i.e., a money-based stabilization), a fully credible exchange rate peg, and the URV. Using a coordination device is better for the vast majority of scenarios. If the government has high debt, an exchange rate peg is better than the URV because it has a stronger signal about the future money growth rate and so about taxes. If the government's debt is already close to its target, the URV is significantly superior to a peg.

The quantitative model helps explain why money-based stabilizations, such as Chile in 1975 and Argentina in 2024, are associated with larger short-run output losses and slower transitions than exchange-rate-based stabilizations, as emphasized by Calvo and Végh (1999). Under a money-based stabilization, firms gradually learn that the transition to low inflation is taking place and continue raising prices faster than money growth declines. This reduces output and remonetization revenue, forcing the government to increase distortionary taxes. Under an exchange-rate-based stabilization, firms set prices in foreign currency and recognize that maintaining the peg requires the monetary authority to reduce money growth. Beliefs, then, converge faster, which leads to a faster fall in inflation and a lower output cost.

In both models, I assume that low inflation is fiscally feasible. The regime-learning literature explains why sustaining low inflation requires fiscal backing. Marcet and Nicolini (2003) show that changes in beliefs can temporarily reduce inflation, but only a decline in average seigniorage makes stabilization permanent. Sargent, Williams, and Zha (2009) extend this logic by allowing monetized deficits to follow latent regimes and distinguishing fundamental fiscal reforms from expectations-driven or cosmetic stabilizations. In my quantitative model, how quickly firms learn about the new regime is crucial for determining the output cost of the transition to low inflation. I study the role of a coordination device in this transition. By using a coordination device, the monetary authority substantially lowers the cost of not learning about the new regime instantaneously, since firms would not have to factor these beliefs in when setting their prices.

A more recent literature on monetary and fiscal interactions develops another mechanism for learning about the fiscal state under rational expectations. In this literature, beliefs about

monetary–fiscal regimes determine whether government liabilities will ultimately be backed by future surpluses or inflation (Bianchi and Ilut, 2017; Bianchi and Melosi, 2019, 2022). The closest to my paper is Bianchi and Ilut (2017), where beliefs about fiscal backing shape the real effects of monetary policy. My paper takes fiscal backing as given and asks what can still prevent disinflation once low inflation is fiscally feasible. The question I ask is why governments repeatedly rely on a coordination device even after fiscal reform, not why inflation persists when there is no fiscal reform.

The inertial-inflation literature shows that lagged indexation and staggered wage and price adjustment can generate persistent high inflation, so a monetary contraction may reduce output before it reduces inflation (Bacha and Lopes, 1979; Lopes and Williamson, 1980; Lopes and Modiano, 1982; Kiguel and Liviatan, 1992b). This concern helps explain why Latin American stabilization plans from the 1960s through the 1990s often combined fiscal adjustment with nominal anchors or temporary incomes policies designed to coordinate wage and price setting (Dornbusch and Fischer, 1986; Dornbusch, 1987; Kiguel and Liviatan, 1989, 1992b).[3] My paper is particularly related to Arida and Resende (1985), who first developed the idea that would become the URV. In the "Larida" framework, the URV mainly removes the inertia generated by lagged and staggered indexation. In my model, the URV changes how firms set prices by choosing their real prices instead of their nominal prices. This allows me to study how the URV, and nominal anchors more generally, affect price-setting behavior and the transition out of hyperinflation. In an extension of my analytical model, I show that with price indexation, the output cost of transitioning to the lower inflation steady state is even higher.

Finally, a related open-economy monetary literature studies the consequences of alternative stabilization instruments once stabilization is fiscally feasible (Rodriguez, 1982; Calvo and Végh, 1992, 1994b; Rebelo and Végh, 1995; Mendoza and Uribe, 1996; Uribe, 1996, 1997; Gregorio, Guidotti, and Végh, 1998). This literature builds models to match the evidence that exchange-rate-based stabilizations tend to generate an initial boom whereas money-based stabilizations tend to generate an initial recession (Végh, 1992; Calvo and Végh, 1994a, 1999; Reinhart and Vegh, 1994).[4] In the canonical imperfect-credibility mechanism of Calvo and Végh (1994a), an expected future abandonment of the exchange-rate peg changes the intertemporal price of consumption and generates an initial spending boom. Other models obtain slow adjustment through staggered wages, backward-looking inflation, currency substitution, durable-goods purchases, or transaction costs. In a quantitative analysis of all

[3]The baseline analysis abstracts from lagged indexation and isolates imperfect common knowledge as a separate source of transition costs. Appendix E.2 adds inherited indexation contracts and shows that they deepen the output cost of disinflation.

[4]Money-based stabilizations are also referred to as cold-turkey disinflations.

possible channels, Rebelo and Végh (1995) emphasize the importance of efficiency gains from lower inflation. My paper asks a different question. Rather than taking the implementation of the stabilization plan as given, I study why the choice of nominal anchor can determine whether stabilization is implemented at all.

Section 2 describes the Brazilian Real Plan, Section 3 develops the model, and Section 4 compares stabilization with and without a coordination device. In Section 5, I build a quantitative model and estimate the output cost associated with the transition to a low-inflation steady state. Section 6 concludes. Appendix E extends the analytical model to allow varying money velocity and lagged indexation.

2 Brazilian Case

The Brazilian Real Plan is ideal for studying the separate roles of a fiscal reform and a coordination device. While most stabilization plans announce the coordination device at the same time as the fiscal reform, the Real Plan was deliberately organized into two phases. In particular, its major fiscal package was in place months before hyperinflation ended. The coordination device, the URV, was introduced only after the government regarded the fiscal prerequisites for stabilization as satisfied.

2.1 The Real Plan

Brazil experienced one of the longest high-inflation episodes in history, lasting more than fifteen years (Fischer, Sahay, and Végh, 2002). After more than a decade of annual inflation above 1,000% and six failed stabilization attempts, the Real Plan finally put an end to the Brazilian hyperinflation.[5] The plan began when Fernando Henrique Cardoso became finance minister in May 1993.

Cardoso identified two causes of hyperinflation: one fiscal and one inertial. Brazil had a serious fiscal imbalance, heavily relying on high inflation to lower its real spending commitments. The government recorded an average primary surplus of about 2% of GDP between 1991 and 1993 in part because inflation eroded the real value of expenditures before they were paid. Between 1990 and 1992, realized expenditure averaged only about 55% of budgeted expenditure in dollar terms, while realized revenue averaged roughly 85% (Franco, 1995). Inflation was thus doing part of the fiscal adjustment by compressing real government spending.[6] Even with this compression, the 12-month operating deficit reached almost 3% of GDP

[5] Ayres et al. (2019) provide a broader overview of Brazil's monetary and fiscal history and discuss the earlier stabilization plans in greater detail.

[6] For a more detailed discussion and a model, see Bacha (1994).

in the second half of 1992, and seigniorage revenue was about 3% of GDP. Attempting to end hyperinflation without a fiscal adjustment would remove both the mechanism compressing real expenditure and the inflationary revenue financing the remaining deficit. The resulting deterioration in the fiscal balance would instead worsen hyperinflation.

The inertial root of hyperinflation was associated with widespread indexation and the front-loading of price and wage adjustments. In hyperinflation, the real value erodes very fast between adjustment dates. For this reason, workers bargained for real wage increases to offset past inflation and preserve purchasing power until the next adjustment. Firms used a similar approach when setting prices. They overshot their real prices to restore margins lost to past inflation and to anticipate the higher wages, input costs, and prices they expected before their next opportunity to adjust (Pazos, 1972; Lopes, 1985; Bresser Pereira and Nakano, 1987).[7] Indexation formalized this behavior by mechanically readjusting wages, prices, and contracts to past inflation.

These roots were addressed in sequence in the Real Plan. Phase I was a fiscal reform that eliminated the government's dependence on inflation to compress real expenditures and finance its deficit. Phase II used a monetary reform to coordinate prices and wages towards low inflation.

2.2 Real Plan Phase I – The Fiscal Reform

The first step to end hyperinflation was to implement a fiscal reform. Phase I took place from the moment Cardoso stepped in as Finance Minister, in May 1993, to March 1994, when the URV was launched. It successfully reduced government spending and increased revenue.

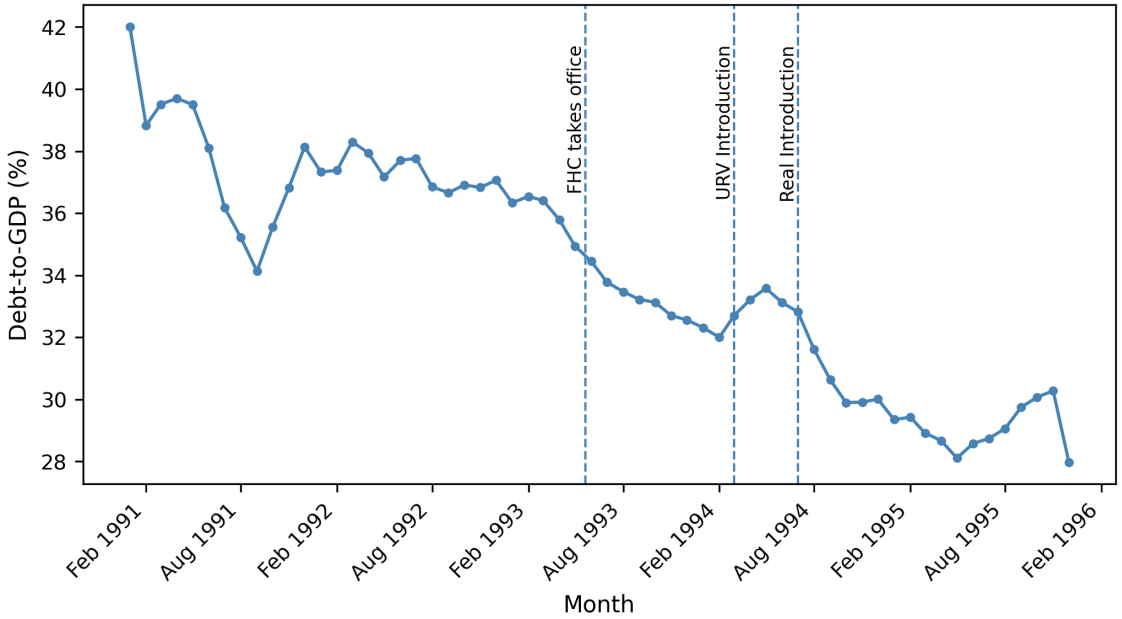
Phase I combined measures to raise revenue, reduce spending commitments, and constrain state-level budgets. The Immediate Action Plan (Programa de Ação Imediata, PAI), announced in June 1993, included the renegotiation of state debt, the privatization of several state-owned local banks, and measures against tax evasion. The IPMF, a 0.25% tax on financial transactions, was approved in July 1993 and took effect on January 1, 1994. The FSE, approved in February and implemented in March 1994, partially de-indexed government spending from government revenue, increased federal control over expenditure, and made a larger surplus possible. Appendix Table 1 reports the complete legislative and policy chronology.

These measures substantially improved the government's budget. As Figure 1 shows, the 12-month operating balance moved from a deficit of almost 3% of GDP in 1992 to a surplus

[7] Werning and Lorenzoni (2024) develop a model that helps to explain this mechanism.

of about 2% for most of 1994. Between 1993 and 1994, federal real revenue rose by 14%, while real spending fell by almost 5%. Public debt also declined. Figure 2 shows that the debt-to-GDP ratio fell from about 37% in early 1993 to 32% by the introduction of the URV and reached a record low of 28% in 1995.

Figure 2: Debt-to-GDP Ratio



Notes: Total public debt of the consolidated public sector as a percentage of GDP, monthly, January 1991–December 1995. Source: BCB. The ratio falls by roughly 10% over the period, with most of the decline before the monetary reform.

By March 1994, the government had completed the fiscal reforms it regarded as necessary for stabilization. In December 1993, Cardoso released a document titled *Cardoso's Plan*, in which he stated that the monetary reform would begin only after Congress approved the remaining fiscal measures needed to make stabilization fiscally feasible. Phase II began on March 1, on the same day that the FSE took effect. The decision to proceed with the monetary reform indicates that, by then, the government considered the fiscal prerequisites for ending hyperinflation to have been satisfied.[8] The conversion of the URV into the Real on July 1 was not associated with any significant fiscal reform or announcement.

[8]Congress's June approval of the IOF rate increase, which had already been in force since January, was the only later fiscal action in the timeline.

2.3 Real Plan Phase II – The Monetary Reform

Phase II introduced the URV and subsequently converted it into the Real. It began on March 1, 1994, the day after Congress approved the last major fiscal measure that the government regarded as necessary for stabilization. The monetary reform coordinated agents to stop increasing nominal prices, which the fiscal reform could not do.

The monetary reform required a coordination device with two distinct functions. First, it had to deal with contractual frictions. That is, it had to replace staggered, backward-looking indexation with a common contemporaneous indexation rule. By using the same rule to update nominal prices, wages, and contracts, it could align relative prices before the end of hyperinflation, avoiding freezing the heavily distorted price dispersion created by hyperinflation. Second, it had to deal with informational frictions. That is, the coordination device had to provide a stable unit of account in which agents could set real prices and wages without having to forecast other agents' inflation expectations. This nominal anchor would allow firms to choose stable real prices without front-loading expected inflation into their nominal prices. A successful coordination device would then realign relative prices and allow firms to coordinate towards a low-inflation regime. In Appendix E.2, I discuss these two requirements and show that if indexation is widespread, the only way to have a costless disinflation is if the coordination device fulfills these two requirements.

Unlike other countries that used an exchange-rate peg, Brazil used the URV. The URV was a nonphysical unit of account. Prices, wages, and contracts could be set in URV, but all payments were still made in Cruzeiro Real, the old currency. The value of the URV in Cruzeiro Real varied daily according to the official price index. A price set in URV remained constant in real terms, while its value in Cruzeiros Reais was automatically updated daily. The Central Bank also maintained the equivalence of one URV to one US dollar, giving the URV a stable external benchmark without adopting the dollar as Brazil's currency.[9] After four months of this dual-unit system, one URV was converted into one Real.

For firms, URV denomination simplified the pricing problem. A firm setting a price in Cruzeiro Real had to forecast the increase in the aggregate price level before its next adjustment and incorporate that expected inflation into its nominal price. A firm setting a price in URV could instead choose the real price it wanted to maintain. Changes in the aggregate price level were absorbed by the conversion factor. This mechanism allowed firms' real prices to be separated from their beliefs about the path of nominal prices and about other firms' responses to stabilization. This separation is the core of how firms can coordinate

[9]Every day, the Central Bank intervened in the foreign exchange market to make sure that the Cruzeiro-Real/Dollar exchange rate was equivalent to the Cruzeiro-Real/URV conversion factor. That worked as a "real exchange-rate peg".

towards low inflation without an abrupt, simultaneous adjustment in nominal prices.

Conversion to the URV was gradual and initially voluntary. Firms were at first required to display prices in Cruzeiro Real, while URV prices were optional.[10] Only after two months were firms allowed to display prices exclusively in URV. This long transition period gave firms time to correctly identify their real prices in URV before it became Real. An immediate switch from Cruzeiro Real to Real would instead have carried the existing dispersion of relative prices into the new currency, potentially leading to massive price adjustments and weakening the stabilization credibility.

Wages and contracts were converted as part of the same process. Wages were converted into URV at their average real value during the preceding four months. This was a compromise between unions and businesses that avoided fixing real wages either at a temporary post-adjustment peak or at the trough reached after inflation had eroded them. Existing contracts could be converted voluntarily. New contracts had to be denominated in URV and could not contain indexation clauses that allowed adjustments more than once a year.

The end of the dual-unit system did not require Brazil to maintain a fixed nominal exchange rate. On July 1, the Central Bank withdrew from the foreign-exchange market even though it was expected to preserve the dollar peg (Franco, 2024). The Real immediately traded at less than one Real per dollar and reached roughly R$0.83 per dollar by October 1994. Franco (2024) argues that this appreciation was important because it increased confidence in the Real without forcing the government to defend a future parity.

Stabilization was followed by a sharp expansion in output. In the third quarter of 1994, real GDP grew by 4.9% relative to the preceding quarter and by 6.0% relative to the same quarter one year earlier. This initial boom is similar to the ones observed after exchange-rate-based stabilizations. It contrasts with the recession that commonly follows money-based programs (Calvo and Végh, 1999). The Brazilian episode shows that avoiding an initial recession did not require a fixed post-stabilization exchange rate when another nominal anchor coordinated price setting.

The URV's role complements the fiscal argument in Sargent (1982). In Sargent's view, a fiscal reform eliminates the need for seigniorage and shifts inflation expectations, allowing hyperinflation to end without output costs. The recurrent use of nominal anchors suggests that fiscal reform alone does not coordinate price setting and inflation expectations quickly enough during the transition. The nominal anchor can give firms a common benchmark for setting prices and forming expectations. The model below formalizes how this coordination device works.

[10]The purpose was to avoid a run into the URV that could intensify hyperinflation in Cruzeiro Real (Franco, 1995)

3 Model

I build on Woodford (2001) to develop a flexible-price model with strategic complementarity, incomplete common knowledge about the monetary regime, and a monetary authority that chooses whether to stabilize. Taking fiscal feasibility as given, I compare two otherwise identical economies. In the first, firms set prices in domestic currency. In the second, they set prices in a stable unit of account with a common conversion factor, the URV.

3.1 Environment and Policy Regimes

Time is discrete, with three periods, $t \in \{0, 1, 2\}$. Period 0 describes the initial conditions under hyperinflation, the reform takes place in period 1, and it becomes common knowledge in period 2. All the action occurs in period 1, when firms are uncertain about what is going to happen in the economy. For this reason, I focus on it in this section.

The economy contains a continuum of monopolistically competitive firms and a monetary authority. The monetary authority privately observes its type, drawn from a continuous distribution F_θ with support $[\underline{\theta}, \bar{\theta}]$, which is common knowledge. In period 1, it chooses onece and forever between two nominal money growth rate regimes:

$$g_t \in \{g^H, g^L\}, \qquad g^H > g^L = 0, \tag{1}$$

where $g_t \equiv \log\left(\frac{M_t}{M_{t-1}}\right)$. Regime g^H corresponds to continued high inflation and g^L to stabilization.

I assume constant money velocity $V > 0$, so choosing the money growth rate is equivalent to choosing the growth rate of nominal aggregate expenditure, $P_t Y_t$, through $M_t V = P_t Y_t$. The economy starts with output at its natural level $Y_0 = \bar{Y}$. I choose the initial currency unit so that $P_0 = 1$. Define inflation by $\pi_t \equiv \log\left(\frac{P_t}{P_{t-1}}\right)$.

The timing is as follows. The initial aggregate price level and money stock are publicly observed before the reform. In period 1, the monetary authority chooses the regime, after which firms receive private signals and set prices. Firms' choices determine the aggregate price level and, through the quantity equation, aggregate output and inflation. This outcome is anticipated by the monetary authority when choosing the regime.

We can rearrange the quantity equation to show that the money growth rate in period t goes to either changes in prices or changes in output:

$$g_t = \pi_t + \Delta y_t, \tag{2}$$

where $y_t \equiv \log\left(\frac{Y_t}{\bar{Y}}\right)$ is the output gap, and $\Delta y_t \equiv y_t - y_{t-1}$ is the change in the output gap. The change in inflation or in the output gap following a non-zero money growth rate depends on the beliefs firms have about what other firms will do.

3.2 Firms and Domestic-Currency Price Setting

I first characterize domestic-currency price setting, where firms can reset prices freely. The only friction is that they do not share common knowledge of the monetary authority's choice in period 1. I index firms by $i \in [0,1]$ and take aggregate output to be a CES composite of differentiated goods with elasticity of substitution $\eta > 1$. Demand for good i is

$$Y_t(i) = Y_t \left(\frac{P_t(i)}{P_t}\right)^{-\eta},$$

where Y_1 is aggregate real expenditure and

$$P_t = \left(\int_0^1 P_t(i)^{1-\eta}\, di\right)^{\frac{1}{1-\eta}}$$

is the corresponding aggregate price index. Firm i takes P_1 and Y_1 as given when choosing its price.

Real profits are

$$\Pi_t(P_t(i); P_t, Y_t) = Y_t \left(\frac{P_t(i)}{P_t}\right)^{1-\eta} - C\left(Y_t \left(\frac{P_t(i)}{P_t}\right)^{-\eta}; Y_t\right).$$

The first term is real revenue. The first argument of $C(\cdot;\cdot)$ is the firm's own output, and the second is aggregate output, which generates strategic complementarity in pricing. Conditional on its private information, firm i solves

$$\max_{P_t(i)} \ \mathbb{E}_t^i \left[\Pi_t(P_t(i); P_t, Y_t)\right],$$

where $\mathbb{E}_t^i[\cdot]$ denotes expectations conditional on firm i's information in period 1. The first-order condition is

$$\mathbb{E}_t^i \left[\frac{\partial \Pi_t(P_t(i); P_t, Y_t)}{\partial P_t(i)}\right] = 0. \tag{3}$$

Under full information, all firms choose the same price, so $P_t(i) = P_t$. I define the natural level of output $\bar{Y}$ as the level in the full-information symmetric equilibrium. Log-linearizing

(3) around the full-information equilibrium gives

$$p_t(i) = \mathbb{E}_t^i[\pi_t] + \xi\, \mathbb{E}_t^i[y_t], \tag{4}$$

The pricing equation has a simple interpretation. Firm i sets its nominal price based on its beliefs about the expected inflation, with an additional adjustment for its beliefs about the expected output gap. This second term captures how the firm's desired relative price responds to aggregate economic activity.[11]

Using $g_t = \pi_t + \Delta y_t$, in (4), firm i's pricing rule becomes

$$p_t(i) = \xi \left(\mathbb{E}_t^i[g_t] + y_{t-1}\right) + (1-\xi)\mathbb{E}_t^i[\pi_t]. \tag{5}$$

Equation (5) separates the two objects in the firm's decision. The first term is the response to the expected current money growth rate, added to any incorrect beliefs the firm had about the output gap in the last period, since the output gap becomes common knowledge one period later. The second is its response to the inflation it expects other firms to generate in this period (i.e., what it believes other firms believe the money growth rate will be). The latter matters for the pricing decision only because of strategic complementarities in the cost function. When ξ is small, strategic complementarities are high, and the firm's desired price is more sensitive to what it expects other firms to do.

To determine inflation and the output gap, we need to find the higher-order beliefs. I define higher-order average expectations recursively by

$$\vartheta_t^{(0)} \equiv \vartheta_t, \qquad \vartheta_t^{(k)} \equiv \int_0^1 \mathbb{E}_t^i \left[\vartheta_t^{(k-1)}\right] di, \qquad k \geq 1. \tag{6}$$

Under this notation, $\vartheta_1^{(k)}$ is the k-th order belief about the variable ϑ in period 1.

The log-linearized CES price index implies

$$\pi_t = \int_0^1 p_t(i)\, di. \tag{7}$$

Since $y_0 = 0$, averaging (5) across firms in period 1 gives us

$$\pi_1 = \xi g_1^{(1)} + (1-\xi)\pi_1^{(1)}. \tag{8}$$

[11] Appendix C.1 derives ξ from the primitives of the cost function. Following Woodford (2001), I assume $0 < \xi < 1$. The restriction $\xi > 0$ implies that a firm raises its desired relative price when it expects aggregate output to be high. The restriction $\xi < 1$ implies strategic complementarity.

Inflation depends on two objects. The first is firms' average expectation of the current money growth rate, which captures their reaction to the policy choice. The second is their average expectation of the inflation generated by other firms, which captures their beliefs about what other firms are doing. Because inflation itself depends on firms' expectations of money growth and of other firms' prices, this second term introduces higher-order beliefs about the money growth rate. Since $\xi \in (0,1)$, iterating (8) yields

$$\pi_1 = \sum_{k=1}^{\infty} \xi(1-\xi)^{k-1} g_1^{(k)}. \tag{9}$$

First-order beliefs receive weight ξ, second-order beliefs receive weight $\xi(1-\xi)$, and so on. Stronger strategic complementarity, corresponding to a smaller ξ, means firms place more weight on higher-order beliefs. The intuition is that when strategic complementarities are higher, each firm's price responds more strongly to what it expects other firms to do.

Using $y_1 = g_1 - \pi_1$ and the fact that the weights sum to one, aggregate output can be written as

$$y_1 = \sum_{k=1}^{\infty} \xi(1-\xi)^{k-1} \left(g_1 - g_1^{(k)} \right). \tag{10}$$

Equations (9) and (10) are key to understanding the role of higher-order beliefs in the model. When common knowledge makes $g_1^{(k)} = g_1$ at every order, inflation absorbs current money growth one for one, and output remains at its natural level. Under imperfect common knowledge, output depends on the gaps between the realized current money growth rate and successive orders of average beliefs. The intuition is that firms will not respond as much to their private signals because they know that other firms' private signals matter for determining the aggregate variables. So, posteriors are attenuated towards the prior belief, which means regime shifts are not fully internalized.

3.3 Information Structure

Firms do not observe the monetary authority's choice directly. In particular, each firm receives a noisy private signal:

$$S_t(i) = g_t + \varepsilon_t(i), \qquad \varepsilon_t(i) \sim N(0, \sigma_g^2), \qquad 0 < \sigma_g < \infty.$$

The noise is independent across firms and independent of the monetary authority's type. Firms share a common prior probability $\lambda_0 \in [0,1]$ that the monetary authority has chosen a low money growth rate. For $\lambda_0 \in (0,1)$, each firm updates its belief about the regime using

Bayes' rule after observing its signal. The posteriors and the aggregation of higher-order beliefs are derived in Appendix C.2.

The signals generate uncertainty about the realized regime and about how other firms interpret it. This information structure and Equation (10) determine the price and output outcomes associated with g^L and g^H. Section 4.1 characterizes those outcomes.

3.4 Monetary Authority

The monetary authority chooses once and forever the money growth rate in period 1. To make this decision, it evaluates each regime choice using the period 1 loss function[12]

$$\mathcal{L}_1(g_1;\theta) = -\theta\, y_1(g_1) + \pi_1(g_1)^2 \qquad \theta \geq 0. \tag{11}$$

A higher θ increases the weight placed on output relative to inflation. The monetary authority evaluates inflation and the output gap in the reform period, $t = 1$. In period 2, the output cost is null because the money growth rate chosen becomes common knowledge.

The loss function is linear in the output gap, so the monetary authority prefers expansions to recessions. This assumption makes the high-money growth rate regime attractive because it generates a boom.

Firms' equilibrium choices define an outcome $\{\pi_1(g_1), y_1(g_1)\}$ under each regime, and the monetary authority compares its losses. The parameter θ determines how strongly the monetary authority weights departures from natural output relative to inflation. Section 4 solves this policy problem under domestic-currency pricing and when firms set prices in URV.

3.5 The URV

Now, I only change the unit in which firms choose prices. I assume their information about the monetary regime remains the same. Under domestic-currency pricing, a firm chooses a nominal price whose real value depends on the aggregate nominal price level. Under the URV, the opposite happens. Firms choose their real price in a stable unit of account, and a common conversion factor gives the nominal price.

Let $X_t(i)$ denote the price that firm i sets in URV and U_t the conversion factor from URV into the nominal unit of payment. The nominal price of good i is

$$P_t(i) = U_t X_t(i), \qquad p_t(i) = u_t + x_t(i).$$

[12] Allowing for an intertemporal loss function would not change the main results and insights. The analytical model would, however, become intractable.

Choose the URV unit so that its initial conversion benchmark is $U_0 = 1$. Define $u_t \equiv \log U_t$ and $x_t(i) \equiv \log X_t(i)$, the firm's log price in URV units. The aggregate price level measured in URV is

$$p_t^U \equiv \pi_t - u_t = \int_0^1 x_t(i)\, di.$$

Define the money growth rate in URV units by

$$g_t^U \equiv g_t - u_t. \tag{12}$$

Using (2), aggregate output in period 1 becomes

$$y_1 = g_1 - \pi_1 = g_1^U - p_1^U.$$

The component of the money growth rate relevant for firms' URV price decisions is the potential gap between the money growth rate and the conversion factor growth, $g_1 - u_1$, rather than g_1 itself.

The historical Brazilian conversion factor tracked the price level, which corresponds to $u_t = \pi_t$ in this setup. However, this rule cannot be imposed directly when all firms set prices in URV. Because aggregate inflation satisfies $\pi_t = u_t + p_t^U$, the conversion factor would be defined through an inflation rate that itself depends on that factor. This generates an indeterminacy problem. For this reason, in the baseline model, I let the URV track directly the money growth rate:

$$u_t = g_t. \tag{13}$$

This rule is common knowledge and is applied after firms choose their URV prices.

The rule is well defined because (1) fixes g_1 independently of firms' current price choices, so the conversion factor never depends on the prices it is used to convert.[13] When output is at its natural level, (2) implies $\pi_1 = g_1$, so a conversion factor that tracks the money stock also grows at the rate of inflation. Section 4.2 characterizes the consequences of this rule for prices, output, and the monetary authority's regime choice.

3.6 Equilibrium

Definition 1 (Equilibrium) *Given money growth rates $g^H > g^L = 0$, a distribution of authority types F_θ, signal noise σ_g, and the unit in which firms set prices, an equilibrium*

[13] Another way to model the URV is to assume that a measure-zero set of firms continue to set prices in domestic currency. Then, one can use the CES price index from these firms as an independent price index, as in Equation (9).

consists of a regime choice $g_1(\theta) \in \{g^H, g^L\}$ *for each authority type, a prior* $\lambda_0 \in [0,1]$*, firms' posterior beliefs and prices* $\{p_1(i)\}_{i\in[0,1]}$*, and an outcome* $\{\pi_1(g_1), y_1(g_1)\}$ *for each regime* $g_1 \in \{g^H, g^L\}$*, such that:*

1. *Firms optimize: given its beliefs, each firm's price satisfies* (4)*. Under the URV, firms choose prices in URV units and nominal prices follow from the conversion rule* (13)*.*

2. *Firms' beliefs are rational: each firm forms its posterior about the regime from the prior* λ_0 *and its private signal by Bayes' rule.*

3. *Markets clear: in each regime, inflation satisfies the price index* (7)*, and output satisfies the quantity equation* (2)*.*

4. *The monetary authority optimizes: for each type* θ*,* $g_1(\theta)$ *minimizes the loss* (11)*, taking as given the outcome under each regime.*

5. *The prior is consistent with the authority's choices:*

$$\lambda_0 = \int \mathbf{1}\{g_1(\theta) = g^L\}\, dF_\theta(\theta).$$

4 Analytical Results

The comparison proceeds in four steps. First, I characterize prices and output when firms set prices directly in domestic currency. Second, I use that outcome to determine which monetary-authority types choose stabilization. Third, I hold fixed the policy set, private signals, prior, and authority objective, but allow firms to set prices in URV. This comparison isolates how the unit of account changes both the real effects of the regime choice and the monetary authority's incentive to stabilize. Fourth, I study the equilibrium beliefs over the monetary authority's choices.

4.1 Without a Coordination Device

Without a coordination device, imperfect common knowledge prevents firms from fully adjusting nominal prices to the realized monetary regime, giving changes in money growth real effects even with flexible prices. The resulting recession under stabilization can make the low-money-growth regime unattractive to sufficiently output-sensitive monetary authorities.

Under domestic-currency pricing, firms must forecast aggregate inflation when choosing their own nominal prices. Equation (10) shows how imperfect common knowledge enters

the resulting level of output. Following a realization of g^L, beliefs continue to place weight on g^H, so inflation falls by less than the reduction in the money growth rate and output contracts. Following a realization of g^H, beliefs continue to place weight on g^L, so inflation rises by less than the increase in the money growth rate and output expands.

Proposition 1 *Fix $\lambda_0 \in (0,1)$ and suppose private signals are noisy and informative. Under imperfect common knowledge in period 1,*

$$y_1^L(\lambda_0) < 0 < y_1^H(\lambda_0), \qquad 0 < \pi_1^L(\lambda_0) < \pi_1^H(\lambda_0) < g^H.$$

If the regime is common knowledge in period 1, then $y_1^K = 0$ and $\pi_1^K = g^K$.

Proof. See Appendix D.1.

Proposition 1 isolates the real effect of imperfect common knowledge. Firms can reset prices freely, but their prices depend on what they expect other firms to believe and do. Under common knowledge, every belief order is the same as the realized regime, so the output gap disappears in both regimes. With imperfect common knowledge, stabilization produces a recession, while continuing with a high money growth rate produces a boom.

The output cost associated with the low money growth rate regime is consistent with the cross-country evidence on money-based stabilizations. Reinhart and Vegh (1994) and Calvo and Végh (1999) show that stabilizations that do not rely on an exchange-rate anchor are typically associated with an initial recession. The baseline economy provides the model counterpart of a money-based stabilization. The monetary authority reduces the money growth rate, but firms embed positive inflation from other firms in their prices because they think other firms will not fully believe in the end of hyperinflation.

There are other stabilization mechanisms that can generate output costs. In standard open-economy monetary models, lower expected inflation raises real money demand. Because the nominal money stock is predetermined and the home-goods price level cannot adjust on impact, real balances cannot increase through lower prices, so output falls to clear the money market (Végh, 1992; Uribe, 1996; Calvo and Végh, 1999). This mechanism assumes that prices are fixed within the period. This assumption is hard to reconcile with hyperinflation, since firms adjust prices very frequently. Incomplete common knowledge provides a more plausible source of incomplete price adjustment. A regime shift is particularly difficult to identify during hyperinflation, when prices and monetary aggregates are changing rapidly. In this scenario, any observation is unlikely to reveal whether the high-inflation regime has actually ended. Even a firm that becomes convinced that policy has changed may not be fully confident that other firms have the same belief. In my model, the output cost comes entirely from this uncertainty about the regime shift and other firms' responses to it.

Models with nominal rigidities usually associate a fully credible disinflation with an output boom. In the Ball (1994) model of staggered prices, a sufficiently gradual and fully credible reduction in the money growth rate can produce a temporary boom. The rationale is that a concave money-stock path induces firms to set prices below the level consistent with constant output, while a declining money growth rate erodes the overhang of fixed prices. Menu-cost models generate long-run gains from disinflation because high inflation creates relative-price dispersion and excessive menu costs (Golosov and Lucas, 2007; Alvarez et al., 2019). In the transition experiment of Beraja (2013), consumption falls slightly on impact and then quickly converges to a higher steady state as the money growth rate declines. I find similar results in Section 5.

The monetary authority is subject to the output effects of its policy choice. For a given prior λ_0, firms' equilibrium beliefs determine the outcomes

$$\left\{\pi_1^L(\lambda_0), y_1^L(\lambda_0), \pi_1^H(\lambda_0), y_1^H(\lambda_0)\right\}.$$

In period 1, the monetary authority has to choose between g^H and g^L. Using its loss function, the monetary authority prefers g^L when

$$\theta \leq T(\lambda_0),$$

where

$$T(\lambda_0) \equiv \frac{\left[\pi_1^H(\lambda_0)\right]^2 - \left[\pi_1^L(\lambda_0)\right]^2}{y_1^H(\lambda_0) - y_1^L(\lambda_0)}.$$

Proposition 1 implies that the denominator is strictly positive. A higher θ makes the monetary authority more reluctant to accept the recession associated with money-based disinflation and more willing to maintain a high money growth rate.

Proposition 2 *Fix $\lambda_0 \in (0,1)$ under imperfect common knowledge in period 1. The monetary authority chooses g^L if and only if*

$$\theta \leq T(\lambda_0).$$

If $T(\lambda_0) < \bar{\theta}$, every type in the support with $\theta > T(\lambda_0)$ chooses g^H. Under common knowledge in period 1, every type chooses g^L.

Proof. See Appendix D.2.

Proposition 2 shows that fiscal feasibility does not by itself implement g^L. Incomplete price coordination can make disinflation suboptimal. Types above the cutoff place sufficient

weight on the recession under g^L and the expansion under g^H to continue with a high money growth rate. Both output gaps are zero under common knowledge, and every type chooses stabilization.

This result is in stark contrast to Sargent (1982) and the traditional prescription for ending hyperinflation: fiscal consolidation is necessary but not sufficient for stabilization. A successful stabilization may also require a device that coordinates price setting and makes the low money growth regime incentive-compatible for the monetary authority. Besides restoring a balanced budget, stabilization plans should address how firms coordinate on the regime change. As I discuss in Section 5, the need for a coordination device is especially strong when information about monetary policy is dispersed, and the regime shift is unlikely to become common knowledge quickly. It is even more important when there is widespread lagged indexation, as I show in Appendix E.2.

Existing explanations for delayed stabilization emphasize political-economy frictions, such as disagreement over who bears the fiscal adjustment or conflict among policymakers (Alesina and Drazen, 1991; Velasco, 1999). Zarazaga (1995) and Mondino, Sturzenegger, and Tommasi (1996) build on related mechanisms to explain recurrent hyperinflation. These theories can explain why inflation persists even when stabilization is fiscally feasible, but the delay arises because political conflict constrains policy. I abstract from these frictions and give the monetary authority full control over the regime choice. Stabilization may still fail because firms' pricing decisions make the low money growth regime recessionary and undesirable for sufficiently output-sensitive monetary authorities.

This mechanism is not unique to hyperinflation, but it becomes much stronger when inflation is high. After any regime change, firms may be uncertain about how quickly other firms will adjust their prices. When inflation is low, this uncertainty requires little front-loading because prices are expected to not change much even if the old regime persists. Under hyperinflation, the difference is much larger. A firm that assigns some probability to continued high inflation has an incentive to keep massively front-loading its price. Setting it for the stabilized regime while other firms continue raising theirs would quickly reduce its relative price. As a result, uncertainty about the regime generates much more aggressive price adjustment and larger output costs under hyperinflation.

The result points to the policy question of what would be a coordination device that can make disinflation incentive-compatible for the monetary authority. In the next subsection, I show that the URV achieves this goal.

4.2 With Coordination Device

I now change only the unit in which firms choose prices. I hold fixed everything else, the monetary authority's policy set and objective, the prior λ_0, and every firm's private signal. Under the URV, the relevant money growth rate object is $g_1^U = g_1 - u_1$, defined in (12). Firms choose their URV prices knowing the state-contingent rule for u_1. The conversion factor is then applied mechanically when nominal prices are determined.

Substituting $p_1(i) = u_1 + x_1(i)$, $\pi_1 = u_1 + p_1^U$, and $y_1 = g_1^U - p_1^U$ into (4) gives the pricing rule in URV units:

$$x_1(i) = \xi\, \mathbb{E}_1^i[g_1^U] + (1-\xi)\mathbb{E}_1^i[p_1^U]. \tag{14}$$

Iterating average expectations as before yields

$$p_1^U = \sum_{k=1}^{\infty} \xi(1-\xi)^{k-1}(g_1^U)^{(k)}, \qquad y_1 = \sum_{k=1}^{\infty} \xi(1-\xi)^{k-1}\left(g_1^U - (g_1^U)^{(k)}\right). \tag{15}$$

The transformed hierarchy has the same structure as under domestic-currency pricing, but it applies to g_1^U rather than g_1. Imperfect common knowledge can therefore affect output only through the component of the money growth rate that the conversion factor leaves unabsorbed.

Under the rule (13), the money growth rate in URV units satisfies

$$g_1^U = g_1 - u_1 = 0$$

under both g^L and g^H. A change in the money growth rate raises the conversion factor one for one and automatically changes every URV-denominated nominal price by the same amount. Firms can choose $x_1(i)$ without forecasting the money growth rate regime, the signals received by other firms, or the nominal prices those firms will post. The conversion factor absorbs all these factors before it becomes relevant for the real-price problem.

Proposition 3 *Suppose firms set prices in URV units. Under either regime $g_1 \in \{g^L, g^H\}$ the unique equilibrium satisfies*

$$x_1(i) = 0, \qquad p_1^U = 0, \qquad y_1 = 0, \qquad \pi_1 = g^K.$$

Inflation is zero under g^L and equals g^H under g^H. Every monetary-authority type chooses g^L.

Proof. See Appendix D.3.

Because $y_1 = 0$ and $\pi_1 = g_1$ under both regimes, the monetary authority's losses are

$$\mathcal{L}_1(g^L; \theta) = 0, \qquad \mathcal{L}_1(g^H; \theta) = (g^H)^2 > 0.$$

Every monetary-authority type therefore chooses g^L, including types that choose g^H under domestic-currency pricing. The URV makes stabilization incentive compatible because a low money growth rate no longer causes a recession, and a high money growth rate no longer creates a boom. This result has the policy-ineffectiveness flavor of Sargent and Wallace (1975), but it operates through the conversion rule, which removes both the recessionary payoff from a low money growth rate and the expansionary payoff from a high money growth rate.

Proposition 3 provides a rational-expectations interpretation of the coordination devices emphasized in the stabilization literature (Dornbusch, 1982; Dornbusch and Fischer, 1986; Dornbusch and Simonsen, 1987; Dornbusch, Sturzenegger, et al., 1990; Lopes, 1985; Bacha, 1987; Kiguel and Liviatan, 1988, 1989, 1992a; Resende, 1988; Edwards, 1996). That literature argues that fiscal adjustment alone does not eliminate inflation because of inertial inflation. Lagged indexation, staggered adjustment, and inflation front-loading carry past inflation into current prices and wages, making a reduction in the money growth rate recessionary. Appendix E.2 adds inherited lagged-indexation contracts and shows that they increase the output cost of disinflation without a coordination device. Bacha (2003, 2024) emphasize the URV role as a common contemporaneous conversion rule. Since it replaced backward-looking indexation, it helped reorganize relative prices around a common unit of account. My proposition offers a complementary rational-expectations interpretation in which the conversion factor removes nominal uncertainty from those decisions. With inherited indexation contracts, eliminating the additional contractual output cost also requires converting lagged contracts into URV.

This result also distinguishes the mechanism from the learning literature. In Sargent, Williams, and Zha (2009), stabilization depends on agents' beliefs about the fiscal regime. However, it does not explain why hyperinflation ends when a nominal anchor is introduced or why such anchors are used so systematically in successful stabilizations. Marcet and Nicolini (2003) argues that introducing a nominal anchor mechanically resets beliefs, which can generate temporary stabilizations. In my model, the URV changes the transition outcome even when firms' beliefs about the monetary regime remain unchanged. The URV eliminates the output cost associated with uncertainty about the monetary regime, which means it affects the transition directly rather than changing the learning rate about the regime.

4.3 Equilibrium Credibility

The previous results take firms' prior probability of stabilization as given. I now make this probability endogenous by requiring it to coincide with the fraction of monetary authority types that choose the low money growth rate.

For a conjectured stabilization probability λ_0, firms' beliefs determine prices and the cutoff $T(\lambda_0)$. Since all authority types with $\theta \leq T(\lambda_0)$ stabilize, $F_\theta(T(\lambda_0))$ is the fraction of types that actually choose the low money growth regime. Equilibrium requires this fraction to validate firms' prior,

$$\lambda_0^* = F_\theta\left(T(\lambda_0^*)\right).$$

Before characterizing the fixed points of this mapping, it is useful to consider what happens at the two extreme priors, $\lambda_0 \in \{0, 1\}$. These cases are particularly interesting since firms are certain about the regime, and still the monetary authority has incentives to deviate from their conjecture.

When $\lambda_0 = 0$, firms are certain that the high money growth regime will continue and set prices accordingly. If the monetary authority unexpectedly chooses g^L, firms do not lower their prices because stabilization had zero prior probability. Inflation remains at g^H on impact and output falls by g^H. Stabilization provides no immediate inflation gain and causes a recession, so every monetary authority type chooses g^H. When $\lambda_0 = 1$, firms are fully certain that stabilization will be chosen and set prices for g^L. If the monetary authority instead chooses g^H, firms cannot raise their prices, and so inflation remains at zero on impact. As a consequence, the entire monetary expansion goes towards increasing output, so every monetary authority type again chooses g^H. The result is that the two extreme cases have the same cutoff, $T(0) = T(1) = 0$, but for opposite reasons. Proposition 4 formalizes it.

Proposition 4 *Suppose $0 < \underline{\theta} < \bar{\theta}$. Under domestic-currency pricing, $\lambda_0^* = 0$ is always an equilibrium, while $\lambda_0^* = 1$ is never an equilibrium.*

Proof. See Appendix D.4.

In these corner cases, incomplete common knowledge disappears, but the credibility problem does not. When $\lambda_0 = 0$ or $\lambda_0 = 1$, all firms are certain about the regime, and all higher-order beliefs coincide. Prices are fully coordinated on that conjectured regime, but this does not make stabilization incentive compatible. The role of incomplete common knowledge appears only for interior priors, where firms are no longer coordinated on a single regime.

In an interior solution, firms assign positive probability to both regimes and $T(\lambda_0) > 0$. Stabilization now lowers inflation relative to g^H, but it still causes a recession. This creates a policy tradeoff and opens up the possibility that stabilization becomes an equilibrium.

Proposition 5 *Suppose F_θ is continuous with support $0 < \underline{\theta} < \bar{\theta}$. Under domestic-currency pricing, any equilibrium other than $\lambda_0^* = 0$ is interior and satisfies*

$$\lambda_0^* = F_\theta\left(T(\lambda_0^*)\right), \qquad 0 < \lambda_0^* < 1.$$

Under the URV, the unique equilibrium has $\lambda_0^* = 1$. Every monetary authority type chooses g^L, and $y_1 = \pi_1 = 0$.

Proof. See Appendix D.5.

An interior equilibrium represents partial credibility. In this case, firms set prices following a weighted average of the optimal price in both regimes. This results in an inflation-output tradeoff that leads low-θ authorities to stabilize and high-θ authorities to maintain high money growth.

Under the URV, the equilibrium structure changes completely. The conversion factor removes the output cost of stabilization and the expansionary payoff from high money growth, so the monetary authority no longer faces a tradeoff between inflation and output when choosing the regime. Stabilization is optimal for every type regardless of firms' prior. The URV makes the low money growth regime incentive compatible and supports full credibility as the unique equilibrium.

The following proposition gives a sharper characterization for the existence of interior equilibria when monetary authority types are uniformly distributed starting at zero.[14]

Proposition 6 *Suppose $\theta \sim U[0, \bar{\theta}]$, with $\bar{\theta} > 0$. Define*

$$\kappa \equiv 2g^H \xi \left[\exp\left(\frac{(g^H)^2}{\sigma_g^2}\right) - 1\right].$$

Under domestic-currency pricing, $\lambda_0^* = 0$ is always an equilibrium. If $\bar{\theta} \geq \kappa$, it is the unique equilibrium. If $0 < \bar{\theta} < \kappa$, there is also exactly one interior equilibrium $\lambda_0^I \in (0, 1)$, characterized by

$$T(\lambda_0^I) = \bar{\theta}\lambda_0^I, \qquad \theta^I = \bar{\theta}\lambda_0^I.$$

Under the URV, the unique equilibrium has $\lambda_0^* = 1$.

Proof. See Appendix D.6.

Proposition 6 shows when a positive-credibility equilibrium can coexist with the zero-credibility equilibrium. The coefficient κ is the slope of the stabilization cutoff at zero. For a small positive prior, $T(\lambda_0)$ is approximately $\kappa\lambda_0$, so the fraction of monetary authority types

[14] With $\theta \sim U[\underline{\theta}, \bar{\theta}]$ and $0 < \underline{\theta} < \bar{\theta}$, there can be zero, one, or two interior equilibria in addition to $\lambda_0^* = 0$.

choosing stabilization is approximately $(\kappa/\bar{\theta})\lambda_0$. If $\kappa > \bar{\theta}$, a small increase in firms' prior generates an even larger increase in the fraction of types willing to stabilize. Near $\lambda_0 = 1$, this cannot continue because the incentive to exploit prices set for stabilization drives the cutoff back to zero. The interior equilibrium lies between these two regions. Since $T(\lambda_0)/\lambda_0$ is strictly decreasing in this case, there is exactly one such equilibrium. If $\kappa \leq \bar{\theta}$, the fraction of types choosing stabilization never becomes large enough to validate a positive prior.

More precise private signals and weaker strategic complementarity raise κ, making an interior equilibrium more likely. A larger $\bar{\theta}$ works in the opposite direction by placing relatively more monetary authority types on high values of output sensitivity. At the interior equilibrium, types below θ^I accept the recession associated with stabilization in exchange for lower inflation, while types above it maintain high money growth. Stabilization can occur with positive probability, but the zero-credibility equilibrium remains.

5 Quantitative Model

In this section, I build a quantitative menu-cost model following Golosov and Lucas (2007) and Alvarez et al. (2019) to analyze the output cost associated with three different disinflation options: money-based, URV, and exchange rate peg. Money-based stabilization has the highest output cost. The exchange rate peg reveals the monetary regime faster, so the government collects the remonetization revenue early, but it exposes prices to exchange-rate noise. The URV does not reveal the regime, but it lets firms set their real prices exactly. The URV has a lower output cost than the exchange rate peg when at least one of the following is true: i) there is no need for a fiscal adjustment; ii) exchange-rate noise is large; and iii) agents learn the regime quickly.

5.1 Environment

Time is continuous and runs forever. The economy is populated by a representative household and a continuum of monopolistically competitive firms indexed by $i \in [0, 1]$. Firms produce differentiated goods and face idiosyncratic and independently distributed cost shocks. Each firm can change its nominal price only by paying a menu cost, paid in units of the consumption good and proportional to its frictionless profit. The model also has a monetary authority, which sets the money growth rate, and a government, which chooses the timing of distortionary labor taxes to bring its debt to a target level.

Aggregate consumption is a CES index,

$$C_t = \left[\int_i C_t(i)^{\frac{\eta-1}{\eta}}\, di\right]^{\frac{\eta}{\eta-1}}, \qquad \eta > 1,$$

with associated price index

$$P_t = \left[\int_i P_t(i)^{1-\eta}\, di\right]^{\frac{1}{1-\eta}}.$$

The core innovation of this model is to introduce incomplete common knowledge about aggregate variables in the context of heterogeneous firms. In particular, I assume firms receive private signals about the money growth rate and the tax rate. Similar to the analytical model, they form beliefs about what state of the world they are in, but they are not sure which one is the true one. Based on their beliefs, they set prices.

5.2 Households

The representative household consumes a bundle of goods, C_t, supplies labor L_t, and holds real balances, $h_t = \frac{M_t}{P_t}$. The representative household has preferences

$$\int_0^\infty e^{-rt}\left[\frac{C_t^{1-\gamma}}{1-\gamma} - \alpha L_t + \chi\frac{h_t^{1-1/\zeta}}{1-1/\zeta}\right] dt,$$

where $\chi > 0$, $\alpha > 0$, $\gamma \geq 1$, and $\zeta > 0$ is the interest elasticity of money demand.

The household holds money and the government's indexed debt b_t. Its budget constraint in real terms is

$$db_t + \frac{dM_t}{P_t} = \left[r_t b_t + (1-\tau_t)\frac{W_t}{P_t}L_t + \Pi_t - C_t\right] dt,$$

where Π_t denotes firms' real profits, W_t is the nominal wage, and r_t is the real interest rate paid on government bonds. The government taxes labor income at rate $0 \leq \tau_t < 1$.

Household optimality gives the demand for good i,

$$C_t(i) = C_t^{1-\gamma\eta}\alpha^{-\eta}(1-\tau_t)^\eta\left(\frac{P_t(i)}{W_t}\right)^{-\eta},$$

Labor supply and money demand give

$$(1-\tau_t)W_t = \alpha P_t C_t^\gamma, \qquad h_t = \left(\frac{\chi C_t^\gamma}{R_t}\right)^\zeta, \tag{16}$$

so that

$$C_t = \alpha^{-1/\gamma} \left(\frac{(1-\tau_t) W_t}{P_t} \right)^{1/\gamma}.$$

Thus aggregate consumption is pinned down by the after-tax real wage. Real balances fall with the nominal interest rate R_t, which is the opportunity cost of holding money. I define the nominal interest rate later, as it depends on beliefs about the money growth rate regime.

5.3 Firms

Each firm produces one good with labor, $Y_t(i) = L_t(i)/Z_t(i)$. $Z_t(i)$ is the idiosyncratic marginal cost of firm i. Its log, $z_t(i) \equiv \log Z_t(i)$, follows $dz_t(i) = \sigma_z \, dB_t^z(i)$, where $B_t^z(i)$ is a firm-specific Brownian motion, independent across firms.

Denote firm i's log markup by $x_t(i) \equiv \log[P_t(i)/(Z_t(i) W_t)]$. If the firm does not change its price,

$$dx_t(i) = -d \log W_t - \sigma_z \, dB_t^z(i). \tag{17}$$

A firm with markup x earns flow profits $\Pi(x; Y_t, w_t, \tau_t)$. $\bar{\Pi}(Y_t, w_t, \tau_t) \equiv \max_x \Pi(x; Y_t, w_t, \tau_t)$ denotes the frictionless profit. If the firm wants to change its price, it has to pay a menu cost given by $c \, \bar{\Pi}(Y_t, w_t, \tau_t)$, with $c > 0$.

The firm does not know the regime. Let λ be its perceived probability of regime L. The firm's problem is to choose the dates $t_1 < t_2 < \dots$ at which it changes its price and the markups x_n it resets to:

$$v_t(x, \lambda) = \max_{\{t_n, x_n\}} \mathbb{E} \Bigg[\int_t^\infty e^{-r(s-t)} \Pi(x_s; Y_s, w_s, \tau_s) \, ds \\ - \sum_n e^{-r(t_n - t)} c \, \bar{\Pi}(Y_{t_n}, w_{t_n}, \tau_{t_n}) \,\Bigg|\, x_t = x, \ \lambda_t = \lambda \Bigg],$$

subject to (17) between price changes. The value function depends on aggregate conditions only through the date t and the belief λ.

In each regime the aggregates follow a deterministic path that the firm knows, so the only uncertainty about them is which regime prevails. The firm weights its profits in the two regimes by λ and $1 - \lambda$. Let

$$\widehat{\Pi}_t(x, \lambda) = \lambda \, \Pi(x; Y_t^L, w_t^L, \tau_t^L) + (1 - \lambda) \, \Pi(x; Y_t^H, w_t^H, \tau_t^H)$$

be expected profits, and define the expected frictionless profit, $\widehat{\bar{\Pi}}_t(\lambda)$, and the expected markup erosion, $\widehat{d \log W_t}(\lambda)$, in the same way.

The value function solves

$$\max\Bigg\{\widehat{\Pi}_t(x,\lambda)+\partial_t v_t+\widehat{d\log W_t}(\lambda)\,\partial_x v_t+\frac{\sigma_z^2}{2}\partial_{xx}v_t+\mathcal{B}_t v_t-rv_t,\\ \max_{x'} v_t(x',\lambda)-c\,\widehat{\bar{\Pi}}_t(\lambda)-v_t\Bigg\}=0, \tag{18}$$

where $\mathcal{B}_t v_t$ collects the terms from the firm's belief updating (see Appendix G.5 for details).

The firm resets to

$$x_t^*(\lambda)=\arg\max_{x'} v_t(x',\lambda),$$

and does so when its markup leaves an inaction region $(\underline{x}_t(\lambda),\bar{x}_t(\lambda))$. At the two thresholds the firm is indifferent between adjusting and waiting: $v_t(x,\lambda)=v_t(x_t^*(\lambda),\lambda)-c\,\widehat{\bar{\Pi}}_t(\lambda)$.

5.4 Monetary Regimes

There are two possible monetary regimes, $K\in\{L,H\}$. Regime H is the high-inflation regime, and regime L is the low-inflation regime. A regime is a rule for the long-term money supply growth rate. In the short term, the monetary authority may deviate from it in case the demand for real balances changes due to a change in the nominal interest rate. In particular, let the long-term money growth rate be given by

$$d\log\bar{M}_t=g^K dt,$$

If the demand for real balances changes, the monetary authority accommodates this change by changing the money growth in that given period. Let $k_t\equiv\frac{h_t}{C_t^\gamma}$ denote the inverse of money velocity. The money supply in period t is then given by

$$M_t=\bar{M}_t\,\frac{k_t}{k_{0^-}}, \tag{19}$$

where k_{0^-} is the value of k_t in the initial stationary equilibrium. On differentiable paths,

$$d\log M_t=g^K dt+d\log k_t.$$

In a stationary equilibrium, k_t is constant and the money stock grows at rate g^K.

The rule pins down nominal spending. Since $M_t=k_tP_tC_t^\gamma$, (19) holds if and only if $P_tC_t^\gamma=\bar{M}_t/k_{0^-}$. By (16), $(1-\tau_t)W_t=\alpha P_tC_t^\gamma$, so

$$d\log W_t=g^K\,dt-d\log(1-\tau_t). \tag{20}$$

The after-tax nominal wage grows at the rate of money growth, whatever happens to money demand.

The nominal interest rate is the real rate plus the long-term money growth rate, $R^K \equiv r + g^K$. If households assign probability $\bar{\lambda}_t$ to regime L,

$$R_t = \bar{\lambda}_t R^L + (1 - \bar{\lambda}_t) R^H = r + \bar{\lambda}_t g^L + (1 - \bar{\lambda}_t) g^H, \tag{21}$$

where $\bar{\lambda}$ is the average belief in the economy.

The nominal rate, and with it money demand, depends on what households believe about the regime, not only on the regime itself. In regime L inflation has ended, but households hold few real balances for as long as they assign weight to regime H. Through k_t, the same is true of the money stock. In regime L money growth is g^L from $t = 0$, but the money stock grows faster than g^L while the nominal rate is falling.

In addition to choosing the money growth rate, the monetary authority can also choose which instrument it is going to use to disinflate the economy. I study three options, $j \in \{N, U, E\}$. If the monetary authority chooses not to use any instrument, it is a money-based stabilization, denoted by $j = N$. If it chooses a URV-like instrument, denote $j = U$. In this case, there is no signal about the regime, but the real prices are exactly as desired. If it chooses an exchange rate peg, $j = E$.

5.5 Government

The government enters with outstanding indexed debt $b_0 > 0$. All debt is denominated in units of the final good, so an unexpected increase in the price level does not reduce its real principal. The government budget constraint is

$$db_t = (r_t b_t + G_t - T_t)\, dt - \frac{dM_t}{P_t}, \qquad T_t = \tau_t w_t L_t, \tag{22}$$

where r_t is the real interest rate on indexed debt and T_t is labor-tax revenue. I assume throughout that government purchases G_t are exogenous and set to zero.

It is important to explain the difference between seigniorage and remonetization revenue. Since $M_t = P_t h_t$, we can rewrite money issuance as

$$\frac{dM_t}{P_t} = \underbrace{\pi_t h_t\, dt}_{\text{seigniorage } s_t dt} + \underbrace{dh_t}_{\text{remonetization}}. \tag{23}$$

Seigniorage is the revenue from replacing the real balances that inflation erodes. Remone-

tization is the revenue from an increase in the demand for real balances, which allows the government to increase the money supply without increasing inflation. Real balances can jump, in which case dh_t is the size of the jump.

The only goal of the government is to bring its debt from b_0 to a target $B \leq b_0$ by date $\bar{t}$, and to keep it there:

$$b_t = B \qquad \text{for all } t \geq \bar{t}. \tag{24}$$

Let $D_t = \exp\left(-\int_0^t r_v dv\right)$. Integrating (22) from 0 to $\bar{t}$ and using (23) gives the intertemporal budget constraint

$$b_0 - D_{\bar{t}} B = \int_0^{\bar{t}} D_t \left[T_t + s_t - G_t\right] dt + \int_0^{\bar{t}} D_t \, dh_t. \tag{25}$$

The government reduces its debt when one of the following revenue sources increases: i) labor taxes; ii) seigniorage; iii) remonetization. The money growth rate is set by the monetary authority and remonetization is determined by the demand for money, so the government can only choose taxes.

The government chooses the path of the labor tax up to $\bar{t}$ to maximize household welfare,

$$\max_{\{\tau_t\}_{t \leq \bar{t}}} \int_0^{\bar{t}} e^{-rt} \left[\frac{C_t^{1-\gamma}}{1-\gamma} - \alpha L_t + \chi \frac{h_t^{1-1/\zeta}}{1-1/\zeta}\right] dt, \tag{26}$$

subject to (25) and $0 \leq \tau_t \leq \bar{\tau}$. It takes as given the monetary regime, the nominal interest rate, and the pricing decisions of firms. At an interior optimum, the welfare cost of raising one more unit of revenue has the same present value at every date before $\bar{t}$, which means the government smooths taxes.

After $\bar{t}$, the tax keeps debt at B. By (22) and (23),

$$\left(T_t + s_t - G_t\right) dt = r_t B \, dt - dh_t, \qquad t \geq \bar{t}. \tag{27}$$

In the stationary equilibrium of regime K, real balances are constant and inflation is g^K, so

$$T^K = rB + G - g^K h^K.$$

The long-run tax depends on the regime, because seigniorage differs across regimes, but not on the instrument chosen by the monetary authority.

5.6 Nominal Pricing, the URV, and the Peg

Besides choosing the money growth rate, the monetary authority also chooses the disinflation instrument. Let $j \in \{N, U, E\}$ denote domestic nominal pricing, URV pricing, and pricing under the peg-style instrument. The URV and an exchange rate peg differ along two dimensions. First, the URV perfectly tracks the price level, whereas the peg does so imperfectly due to noise. Second, the URV can be adopted without any fiscal commitment and thus conveys no information about the monetary authority's chosen regime. In contrast, implementing an exchange rate peg requires full commitment to low money growth, making its adoption a perfect signal of the monetary regime.

The exchange rate peg and the URV work similarly to their counterparts in the analytical model. For either unit-of-account instrument, write

$$P_t(i) = U_t^j X_t^j(i).$$

The firm sets its real price $X_t(i)$ and lets the conversion factor automatically convert it to $P_t(i)$ according to U_t. The markup for firm i when the instrument j is available follows

$$dx_t^j(i) = d\log U_t^j - d\log W_t - dz_t(i).$$

This equation separates movements in the conversion factor from movements in nominal marginal cost.

The URV satisfies

$$U_t^U = P_t, \qquad \frac{P_t(i)}{P_t} = X_t^U(i).$$

Once a firm sets its price in URV, its markup moves with real marginal cost. In particular, its markup law is

$$dx_t^U(i) = -d\log w_t - dz_t(i) = -\gamma\, d\log C_t + d\log(1-\tau_t) - dz_t(i).$$

Under the peg-style instrument, U_t^E is the exchange rate: the domestic-currency value of one unit of the foreign reference currency. The first key difference between the URV and the exchange rate peg is that the peg is not a perfect instrument, as the URV is. Because there may be deviations from the PPP in the short term, I assume that prices set in domestic currency are affected by an iid shock, $\sigma_\varepsilon dB_t^\varepsilon$. This shock makes prices set in the foreign currency not fully track the desired real price. For simplicity, the deviations from the parity follow a random walk:

$$d\varepsilon_t^E = \sigma_\varepsilon dB_t^\varepsilon, \qquad \sigma_\varepsilon > 0.$$

The Brownian innovation B_t^{ε} is common to all firms and is independent of idiosyncratic cost shocks and private signals. The exchange rate is exogenous, while the domestic price index is determined by the posted quotes through the CES price index, $P_t = U_t^E[\int_0^1 X_t^E(i)^{1-\eta} di]^{1/(1-\eta)}$. Hence the real exchange rate U_t^E/P_t is endogenous.

Firms can correct for this noise when resetting a price, but future exchange-rate innovations are unpredictable. Since the exchange rate follows a random walk, for $\Delta > 0$,

$$\mathbb{E}_t[\varepsilon_{t+\Delta}^E] = \varepsilon_t^E,$$
$$\mathrm{Var}_t(\varepsilon_{t+\Delta}^E) = \sigma_\varepsilon^2 \Delta.$$

Substituting the exchange-rate process into the markup law gives

$$dx_t^E(i) = \sigma_\varepsilon dB_t^{\varepsilon} - d\log W_t - dz_t(i).$$

5.7 Information

Firms do not observe the true regime directly. They start at $t = 0$ from a common prior $\lambda_0 \in (0, 1)$ that the regime is L. Firm i's posterior probability that the economy is in the stabilization regime given the regime choice $j \in \{N, U, E\}$ is given by

$$\lambda_t^j(i) = \Pr_i(K = L).$$

For $j \in \{N, U\}$, I represent beliefs in log-odds form:

$$\ell_t^j(i) = \log\left(\frac{\lambda_t^j(i)}{1 - \lambda_t^j(i)}\right), \qquad \lambda(\ell) = \frac{\exp(\ell)}{1 + \exp(\ell)}.$$

The government's instrument choice may reveal information about the money growth rate. In particular, given the full credibility of the exchange rate peg, if the government decides to implement it, I assume that $\lambda_t^E(i) = 1$ for all i and t. This is the second key distinction between the URV and an exchange rate peg. While the former does not give any information about the regime chosen by the monetary authority, the latter gives a perfect signal.

Choosing the URV, U or cutting the money growth rate, N, does not issue any direct

signals. For $j \in \{N, U\}$, private monetary and tax signals follow

$$dS_t^{g,j}(i) = g_K dt + \sigma_g dB_t^{g,j}(i),$$
$$dS_t^{\tau,j}(i) = \tau_{K,t}^j dt + \sigma_\tau dB_t^{\tau,j}(i),$$

The monetary signal is a noisy observation of the long-run money growth rate g_K in the rule (19). The signal innovations are independent of each other, private cost shocks, and the common exchange-rate shock. This is the only way agents extract information about the tax and money growth rate. Any additional inference from aggregate wages, the money stock, transaction prices, and sales is ruled out. In particular, firms do not infer the regime from their own nominal costs, and households do not infer it from the tax rate they pay.

For an interior posterior, Bayes' rule gives

$$d\ell_t^j(i) = \frac{g_L - g_H}{\sigma_g^2}\left[dS_t^{g,j}(i) - \frac{g_L + g_H}{2}dt\right] + \frac{\tau_{L,t}^j - \tau_{H,t}^j}{\sigma_\tau^2}\left[dS_t^{\tau,j}(i) - \frac{\tau_{L,t}^j + \tau_{H,t}^j}{2}dt\right]. \tag{28}$$

Notice that beliefs about the likelihood of a given money growth rate regime involve signals about the money growth rate itself and the tax rate. This is because the government has to pick a tax rate that closes the budget. Agents can therefore use signals about the tax rate to infer which money growth rate the monetary authority chose.

I assume that households price nominal claims with the average belief of firms. In true regime K,

$$\bar{\lambda}_t^{j,K} = \int_0^1 \lambda_t^j(i)\, di, \qquad j \in \{N, U\}, \qquad \bar{\lambda}_t^E = 1. \tag{29}$$

This is the belief that enters the nominal rate (21). At $t = 0$, $\bar{\lambda}_0^{j,K} = \lambda_0$ under N and U. It then converges to one in regime L and to zero in regime H.

5.8 Equilibrium

Definition 2 (Equilibrium) *Given an instrument $j \in \{N, U, E\}$, money growth rates $g^L < g^H$, a prior λ_0, a debt target $(B, \bar{t})$, and initial conditions $(F_0, b_0, \bar{M}_0)$, an equilibrium consists of, for each regime $K \in \{L, H\}$, paths of allocations $\{C_t, L_t, h_t\}$, prices $\{P_t, W_t, R_t, r_t\}$, policies $\{\tau_t, b_t, M_t\}$, firm values and reset markups $\{v_t^j, x_t^{j,*}\}$, beliefs $\{\lambda_t^j(i), \bar{\lambda}_t^{j,K}\}$, and distributions of firms $\{F_t^K\}$ such that:*

1. *Households optimize:* (16) *holds, and the nominal rate satisfies* (21) *at the average*

belief (29).

2. *Firms optimize:* v_t^j *solves* (18) *given the aggregate paths in both regimes, with beliefs that follow* (28) *from* λ_0 *under* N *and* U*, and* $\lambda_t^E(i) = 1$.

3. *Policy: money follows* (19) *with growth* g^K*. Taxes solve* (26) *up to* $\bar{t}$ *and satisfy* (27) *afterwards, with debt given by* (22) *and* (24).

4. *Markets clear: goods,* (66)*; labor,* (65)*; and money,* $M_t = P_t h_t$.

5. *The distribution* F_t^K *evolves from* F_0 *according to firms' pricing policies and the laws of motion of markups, costs, and beliefs in regime* K *(Appendix G.6).*

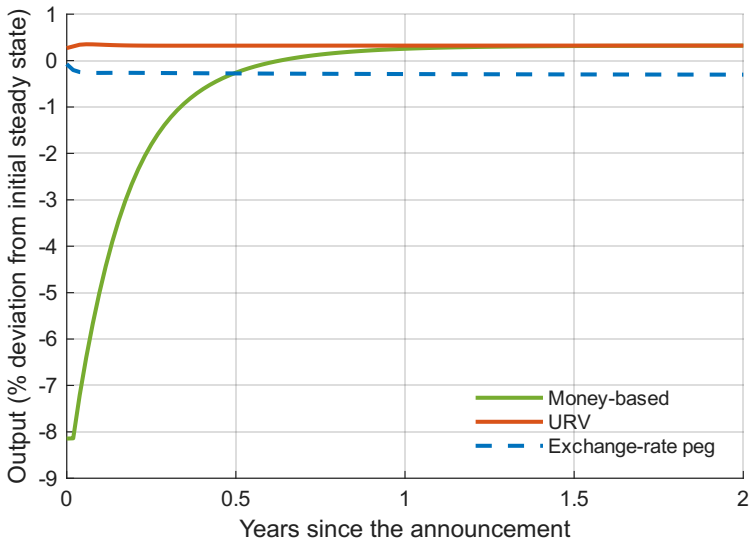
5.9 Disinflation Experiments

I now use the model to measure the output cost of ending a hyperinflation with each of the three instruments. The economy starts in the stationary equilibrium of regime H, with money growth $g^H = 2$, an inflation rate of about 640% a year. At $t = 0$ the monetary authority lowers money growth to $g^L = 0$ and announces the instrument. Under the money-based stabilization and under the URV, agents assign prior probability $\lambda_0 < 1$ to the new regime and learn about it from their signals. Under the peg, the regime is known from the start. Table 2 in Appendix G.10 lists the parameters. All figures show the economy in regime L, in deviation from the initial stationary equilibrium.

No government. The first exercise has no government, as in the analytical model: there is no debt and no labor tax. Figure 3 shows output under the three instruments.

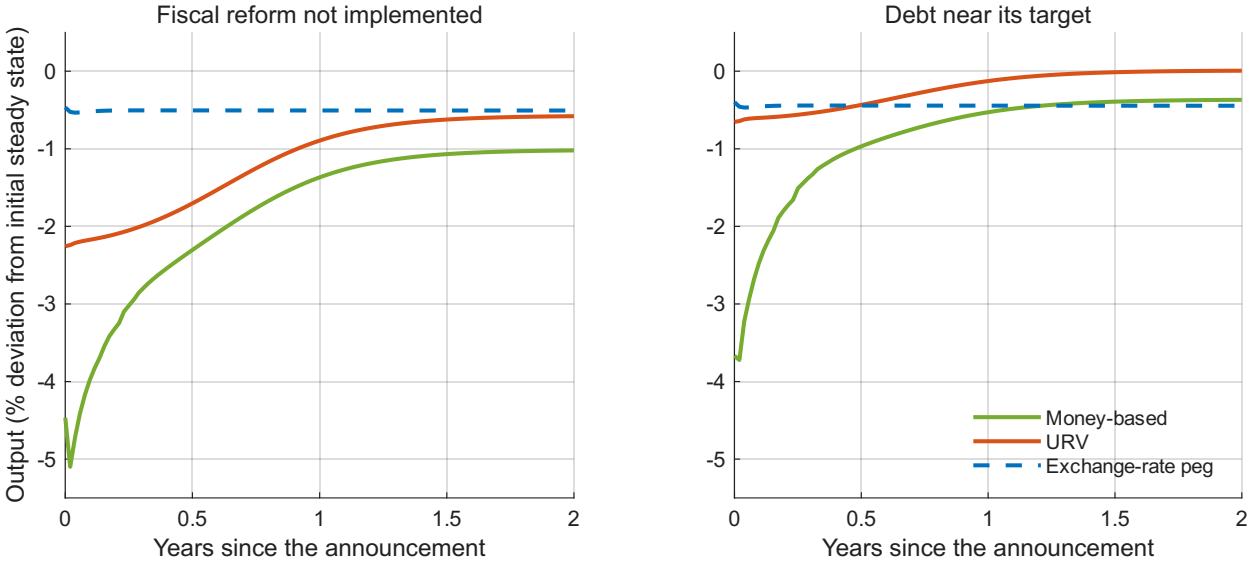
Among the three options, the URV is the one with the lowest output cost. Money-based stabilization has the highest output cost. Output falls by about 8% at the announcement. The nominal wage falls while prices are sticky, since firms are expecting positive inflation, so markups are too high. Under the URV, the disinflation has no output cost. Firms post real prices, so neither the path of the nominal wage nor their beliefs about the regime affect their markups. Under the peg, output does not fall at the announcement either, because nominal prices are readjusted automatically. However, it settles 0.6 percentage points below its level under the URV. The exchange rate adds risk to a firm's markup, so the firm tolerates larger deviations of its markup before it pays the menu cost. This generates a higher misallocation, which permanently lowers output.

Figure 3: Disinflation without a Government



A government with a debt target. The second exercise adds the government of Section 5.5. It must bring its debt from $b_0 = 0.4$, about 85% of annual output, to a target B within five years. Figure 4 compares the output of two economies following the end of a hyperinflation. They differ in how large the fiscal adjustment has to be. In the left-hand side figure, the fiscal adjustment is about 15% of yearly output. In the right-hand side figure, the fiscal adjustment is minimal, less than 1% of yearly output.

Figure 4: Disinflation with a Debt Target: Output



When the fiscal reform has not been implemented, the peg is the better instrument. It reveals the regime, so real balances are rebuilt at the announcement and the remonetization

revenue retires debt at once. The labor tax stays close to its initial level, and output is 0.5% below its initial level, which is the cost of exchange-rate risk. Under the URV, remonetization is slower because it takes time for beliefs to converge, which keeps the amount of money held by households low. Until the remonetization revenue arrives, the government has to raise the labor tax, from 4.2% to 6.6% at the announcement, causing output to fall by 2.2%. The gap to the peg narrows as beliefs converge. The money-based stabilization has the same path of real balances as the URV and adds the cost of incomplete common knowledge to prices.

When debt is near its target, the ranking of the URV and the peg is reversed after the first months. Less debt has to be retired through remonetization, so its timing matters less. Since taxes were already higher, output goes back to where it was during hyperinflation. The money-based stabilization still has the highest output cost, but lower than when a fiscal reform is also needed.

The bottom line of this exercise is that an exchange rate peg might make sense if the government is not in a sound fiscal position and needs quick remonetization revenue to balance its budget. If the government has already implemented most of its fiscal reforms, as was the case in Brazil in 1994 and Argentina in 2024, a URV-like instrument is more suited.

6 Conclusion

This paper shows why fiscal reform is necessary but not sufficient to end a hyperinflation. Motivated by the Brazilian experience, I use incomplete common knowledge and strategic complementarities to identify a belief coordination problem that arises during the transition to low inflation. I study the Brazilian URV as a coordination device that removes this problem and allows stabilization to occur without an output cost. In a quantitative menu-cost model, I show that the URV has the lowest output cost when the country does not need a fiscal reform. If the country needs one, an exchange rate peg generates a lower output cost.

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A Additional Historical Evidence on the Real Plan

Table 1: Timeline of Events Leading to the Real Plan

Date	**Event**
December 29, 1992	Itamar Franco takes office as President of Brazil after Collor is impeached.
May 21, 1993	Cardoso takes office as Finance Minister with the goal of fighting hyperinflation.
June 14, 1993	The Immediate Action Plan (Programa de Ação Imediata, PAI) is released.
June 24, 1993	Itamar Franco signs a bill indexing the minimum wage each month to the preceding month's inflation.
July 13, 1993	The IPMF tax on financial transactions is created.
August 19, 1993	Congress approves a minimum-wage bill setting the adjustment at 90% of the preceding month's inflation.
Interim period	Other minor bills concerning tax revenue and government spending are passed.
December 7, 1993	Cardoso releases the document known as the Cardoso Plan.
December 29, 1993	The IOF tax rate on financial transactions is increased.
January 1, 1994	The IPMF is implemented.
January 28, 1994	Income-tax rates and other corporate taxes are increased.
March 1, 1994	The FSE is implemented, de-indexing part of government spending from government revenue.
March 1, 1994	The URV is introduced, with the provision that it become the new currency, the Real, within at most one year.
June 21, 1994	Congress approves the IOF increase.
July 1, 1994	The URV is converted into the Real.

B Discussion

B.1 Alternative Coordination Devices

The URV is not the only device that can coordinate a disinflation. This section compares it with price-freezing incomes policies, dollarization, a currency board, and a credible exchange-rate peg.

Price-freezing incomes policies coordinate disinflation by fixing wages and prices. In the model, this is equivalent to constraining the nominal price schedule $\{P_t(i)\}_{i\in[0,1]}$. This means that the economy inherits the relative-price configuration that happens to be in place when the freeze begins. Brazil tried this approach with the Cruzado Plan in 1986. Its failure shows the main problem with price freezing. Some prices were below equilibrium when the freeze was imposed, while others were above it. But because keeping prices frozen was crucial to the plan, the relative-price changes necessary to correct those distortions or to respond to later shocks became impossible. Goods with prices that were too low faced excess demand and rationing, while goods with prices that were too high faced weak demand.

The URV avoids this problem. Policy determines only the common conversion factor u_t, while each firm is free to choose $x_t(i)$. The credibility of the stabilization is on the common conversion rule u_t, not on keeping firms' prices fixed, as under a price freeze. Firm-specific prices, x_t^i, can adjust without threatening the credibility of the stabilization. The URV coordinates the common nominal component of prices without requiring the government to determine the correct set of relative prices.

Dollarization and a credible exchange-rate peg can play a similar role to the URV. Firms can set prices in a stable foreign currency even when payments are made in domestic currency. If the exchange rate is expected to remain fixed, it works as a common rule for converting those prices into domestic currency. Firms no longer need to forecast the domestic price level separately or anticipate how other firms will respond to stabilization. The coordination logic is the same as the one behind the URV.

The dollar was a natural alternative unit of account for Brazil, but the URV had important advantages. Firms could have posted prices in dollars while continuing to accept payment in Cruzeiro Real. However, a fixed dollar price corresponds to a stable real price in domestic currency only if movements in the exchange rate only track domestic inflation, effectively requiring relative purchasing-power parity at a very high frequency. Capital flows, speculation, and other shocks are important sources of exchange rate movements, but are independent of domestic prices. When these other factors cause exchange rate movements, the real value of a fixed dollar price fluctuates. The URV avoided this problem because its

conversion factor only tracked domestic prices, so a fixed URV price could remain stable in real terms without relying on the exchange rate. Section 5 develops a quantitative model to evaluate the trade-offs between an exchange rate peg and the URV.

A more radical alternative for Brazil was full dollarization, meaning using dollars as a means of payment, not just as a unit of account. This would have required enough foreign currency to accommodate the sharp increase in money demand that typically accompanies stabilization and remonetization. It would also have eliminated monetary autonomy and faced legal and political obstacles. Another important risk was accelerating the flight out of the Cruzeiro Real before stabilization was complete. The URV required a much smaller institutional change. It changed the unit of account while leaving the domestic currency as the means of payment, avoiding these costs (Franco, 1995, 2000, 2024).[15]

A credible exchange-rate peg or currency board can provide the same type of anchor while preserving the domestic means of payment. In the model, the result holds when firms trust the parity. The end of the Bolivian hyperinflation illustrates it. Before stabilization, the dollar was already used as a unit of account and store of value, while most transactions were still made in pesos (Sachs, 1987; Morales and Sachs, 1990). When the central bank managed to stabilize the exchange rate, it automatically stabilized the peso value of prices set in dollars. A crucial part of these exchange-rate-based instruments is full credibility. As soon as there is uncertainty about the exchange rate, firms price in the possibility of a devaluation and must again form beliefs about how other firms will respond, bringing back the coordination problem.

For this reason, a peg imposes stricter conditions than the URV. For example, if domestic inflation remains above foreign inflation after the peg is introduced, the real exchange rate appreciates and pressure for a devaluation increases. Once firms begin to price in that possibility, a dollar price no longer provides a stable mapping into domestic currency because it also reflects exchange-rate risk. In addition to that, maintaining the anchor also requires the central bank to have sufficient reserves to defend the parity against capital flows and speculative pressure, which is costly.

When compared to a peg, the requirements for the URV to work are much narrower. The URV only requires firms to trust that the conversion factor tracks the economy's price level, such that real prices are stable. There is no requirement about the exchange rate fluctuations, the amount of foreign reserves, or speculative attacks. It is true that the URV makes fiscal and monetary policy irrelevant because it automatically absorbs any changes in

[15]Caravello, Werning, and Martinez-Bruera (2023a) show that anticipated dollarization can cause an immediate devaluation and higher inflation before conversion, whereas Caravello, Werning, and Martinez-Bruera (2023b) show that dollar scarcity can produce an initial recession followed by recovery and transitional inflation.

the money growth rate. However, dollarization, a currency board, or an exchange rate peg constrains monetary and fiscal policies in the same way.

The URV is also easier to remove from the economy. The URV was in place in Brazil for four months, and that was enough to end hyperinflation. Argentina had a currency board for more than ten years, and Mexico had an exchange rate peg for almost that long. The problem is that policymakers do not want to remove the peg or the convertibility because they fear that once they do it, inflation will increase substantially because agents will stop believing in the currency. That indeed happened in Argentina and Mexico. When Argentina abandoned its currency board in 2002, annual inflation reached 42%. After Mexico abandoned its exchange-rate peg in the final days of 1994, annual inflation reached 52% in 1995. For all those years, while the currency could be gaining agents' confidence, it was instead importing confidence from the dollar. Once the peg was over, confidence went away. In Brazil, the central bank introduced a crawling peg almost a year after the end of hyperinflation, which lasted for a bit less than four years.

B.2 Fiscal Feasibility and Credibility

There is no consensus that the fiscal reforms of the Real Plan were durable enough, on their own, to deliver the fiscal adjustment needed to end hyperinflation. The plan's architects instead view them as the beginning of a broader reform process. Fiscal consolidation was still incomplete when inflation collapsed, but the initial success of the plan made further reforms politically feasible later in the decade (Ayres et al., 2019; Portugal, 2016; Bacha, 2024).

There is another debate about whether the fiscal reform was large enough by the time inflation collapsed. Giambiagi (1996) shows that the size of the fiscal adjustment at the yearly frequency was not very different from measures adopted in earlier failed plans. Portugal (2016) emphasizes that much of the 1994 surplus reflected a temporary restriction on spending. Because Congress did not approve the 1994 federal budget until November, monthly expenditure remained capped at 1/12 of the proposed 1994 budget while tax revenue increased. Ayres et al. (2019) also view fiscal reform as important, but insufficient given the fiscal deterioration of the later years. They argue that the plan's initial success and improved access to debt finance helped sustain stabilization.

In Sargent (2013) "A Reasonable Doubt about the Real Plan", Sargent treats the Real Plan's de-indexation as a technical detail that did not address the fiscal cause of inflation. He claimed to be puzzled by the end of the Brazilian hyperinflation given that many relevant fiscal reforms had not yet been implemented. His argument for the success is that monetary

reform bought time for later fiscal adjustment. In other words, the fiscal reform was not fully implemented when hyperinflation ended. In this paper, the URV has a much more relevant role during the transition, while keeping fiscal reform as the condition for durable stabilization.

C Appendix to the Analytical Model

C.1 Derivation of the Pricing Rule

Let $d_t(i) = P_t(i)/P_t$ denote firm i's relative price and let $q_t(i) = Y_t d_t(i)^{-\eta}$ denote its output. Real profits are

$$\Pi_t(d_t(i); Y_t) = Y_t d_t(i)^{1-\eta} - C(q_t(i); Y_t).$$

Because $\partial d_t(i)/\partial P_t(i) = 1/P_t$, the first-order condition for $P_t(i)$ is equivalent to

$$0 = \frac{\partial \Pi_t(d_t(i); Y_t)}{\partial d_t(i)} = Y_t d_t(i)^{-\eta-1} \left[(1-\eta) d_t(i) + \eta C_q(q_t(i); Y_t)\right].$$

Where $C_q(q_t(i); Y_t) = \frac{\partial C(q_t(i); Y_t)}{\partial q_t(i)}$. At the symmetric full-information allocation, $d_t(i) = 1$ and $q_t(i) = Y_t = \bar{Y}$, so the term in brackets must be zero at the optimum. Log-linearizing that term around the symmetric allocation gives

$$0 \approx (1-\eta)\hat{d} + \eta d C_q(q_t(i); Y_t) = (1-\eta)\hat{d} + \eta \bar{Y} C_{qq} \hat{q} + \eta \bar{Y} C_{qY} \hat{y}$$

Define $p_t(i) \equiv \log(P_t(i)/P_{t-1})$. Since $\hat{d} = p_t(i) - \pi_t$ and $\hat{q} = \hat{y} - \eta \hat{d}$,

$$0 = -\left[\eta - 1 + \eta^2 \bar{Y} C_{qq}|_{(q_t(i),Y_t)=(\bar{Y},\bar{Y})}\right](p_t(i) - \pi_t) + \eta \bar{Y}\left[C_{qq}|_{(q_t(i),Y_t)=(\bar{Y},\bar{Y})} + C_{qY}|_{(q_t(i),Y_t)=(\bar{Y},\bar{Y})}\right]\hat{y},$$

It follows that $p_t(i) = \pi_t + \xi y_t$, with

$$\xi \equiv \frac{\eta \bar{Y}\left[C_{qq}|_{(q_t(i),Y_t)=(\bar{Y},\bar{Y})} + C_{qY}|_{(q_t(i),Y_t)=(\bar{Y},\bar{Y})}\right]}{\eta - 1 + \eta^2 \bar{Y} C_{qq}|_{(q_t(i),Y_t)=(\bar{Y},\bar{Y})}}.$$

Under imperfect information, taking conditional expectations of the log-linearized first-order condition gives (4).

C.2 Signal Extraction

In period t, firms do not observe g_t directly. Firm i receives the private signal

$$S_t(i) = g_t + \varepsilon_t(i), \qquad \varepsilon_t(i) \sim N(0, \sigma_g^2),$$

with idiosyncratic noise independent across firms. Let $\lambda_{t-1} \in (0,1)$ denote the prior probability that $g_t = g^L$. The conditional signal densities are

$$f_H(z) = \frac{1}{\sigma_g}\phi\left(\frac{z - g^H}{\sigma_g}\right), \qquad f_L(z) = \frac{1}{\sigma_g}\phi\left(\frac{z - g^L}{\sigma_g}\right),$$

where ϕ is the standard normal density. Firm i's posterior probability that the low-money growth rate regime has been chosen is

$$\lambda_t(S_t(i); \lambda_{t-1}) = \frac{\lambda_{t-1} f_L(S_t(i))}{(1-\lambda_{t-1}) f_H(S_t(i)) + \lambda_{t-1} f_L(S_t(i))}. \tag{30}$$

C.3 Higher-Order Beliefs

Define the average first-order belief that $g_t = g^L$ under each realized regime as

$$\alpha(\lambda_{t-1}) \equiv \mathbb{E}\left[\lambda_t(S_t(i); \lambda_{t-1}) \mid g_t = g^H\right],$$
$$\beta(\lambda_{t-1}) \equiv \mathbb{E}\left[\lambda_t(S_t(i); \lambda_{t-1}) \mid g_t = g^L\right],$$

and let

$$\rho(\lambda_{t-1}) \equiv \beta(\lambda_{t-1}) - \alpha(\lambda_{t-1}).$$

Because $g^H > g^L$ and signals have finite Gaussian noise, the posterior probability of the low regime is strictly decreasing in the signal. Hence,

$$0 < \alpha(\lambda_{t-1}) < \beta(\lambda_{t-1}) < 1, \qquad 0 < \rho(\lambda_{t-1}) < 1.$$

Let $\bar{\lambda}^K_{t,k}$ denote the kth-order average belief that $g_t = g^L$ when the realized regime is g^K, for $K \in \{H, L\}$. The first-order belief is the cross-sectional average of firms' posteriors. Each subsequent order is the cross-sectional average of firms' expectations of the preceding order, as defined in (6).

I begin by defining first-order beliefs. Begin by noting that the independence of signals

across a continuum of firms implies that the expectation equals the true value:

$$\bar{\lambda}^H_{t,1} = \alpha(\lambda_{t-1}), \qquad \bar{\lambda}^L_{t,1} = \beta(\lambda_{t-1}).$$

I now use the fact that Bayes' rule implies that the expected posterior equals the prior:

$$\begin{aligned}
&(1-\lambda_{t-1})\alpha(\lambda_{t-1}) + \lambda_{t-1}\beta(\lambda_{t-1}) \\
&\quad = \int \lambda_t(z;\lambda_{t-1}) \left[(1-\lambda_{t-1})f_H(z) + \lambda_{t-1}f_L(z)\right] dz \\
&\quad = \lambda_{t-1} \int f_L(z)\, dz = \lambda_{t-1}.
\end{aligned}$$

Using the equation above and $\beta = \alpha + \rho$, we obtain the first-order average unconditional belief:

$$\begin{aligned}
\bar{\lambda}_{t,1} &= (1-\lambda_{t-1})\bar{\lambda}^H_{t,1} + \lambda_{t-1}\bar{\lambda}^L_{t,1} \\
&= (1-\lambda_{t-1})\alpha(\lambda_{t-1}) + \lambda_{t-1}\beta(\lambda_{t-1}) \\
&= \alpha(\lambda_{t-1}) + \rho(\lambda_{t-1})\lambda_{t-1}.
\end{aligned}$$

At the firm level, the reasoning is as follows: The firm observes its private signal, $S_t(i)$, and forms a posterior belief about the regime chosen by the monetary authority, $\lambda^1_t(S_t(i);\lambda_{t-1})$. To form its second-order belief, the firm must infer what other firms believe. It does not observe their signals, but it knows how their average beliefs depend on the realized regime. That is, the firm knows $\alpha(\lambda_{t-1})$ and $\beta(\lambda_{t-1})$ since they are only functions of the prior, which is common knowledge. The firm thus uses its own posterior probability of the low regime to weight the average first-order beliefs under each regime. The resulting second-order belief is

$$\begin{aligned}
\mathbb{E}_{i,t}[\bar{\lambda}_{t,1}] &= \left(1-\lambda_t(S_t(i);\lambda_{t-1})\right)\alpha(\lambda_{t-1}) + \lambda_t(S_t(i);\lambda_{t-1})\beta(\lambda_{t-1}) \\
&= \alpha(\lambda_{t-1}) + \rho(\lambda_{t-1})\lambda_t(S_t(i);\lambda_{t-1}).
\end{aligned}$$

Averaging across firms gives

$$\bar{\lambda}^K_{t,2} = \alpha(\lambda_{t-1}) + \rho(\lambda_{t-1})\bar{\lambda}^K_{t,1}, \qquad K \in \{H, L\}.$$

Because this relation is affine and holds under either realized regime, the same argument applies at every order:

$$\bar{\lambda}^K_{t,k+1} = \alpha(\lambda_{t-1}) + \rho(\lambda_{t-1})\bar{\lambda}^K_{t,k}, \qquad K \in \{H, L\}. \tag{31}$$

Each additional order keeps a fraction $\rho(\lambda_{t-1})$ of the previous order, while the remaining weight goes to the prior. That is, as the belief order increases, the posterior gets attenuated towards the prior.

Iterating (31) yields

$$\begin{aligned}\bar{\lambda}^H_{t,k} &= \alpha(\lambda_{t-1})\frac{1-\rho(\lambda_{t-1})^k}{1-\rho(\lambda_{t-1})},\\ \bar{\lambda}^L_{t,k} &= \alpha(\lambda_{t-1})\frac{1-\rho(\lambda_{t-1})^k}{1-\rho(\lambda_{t-1})} + \rho(\lambda_{t-1})^k.\end{aligned} \tag{32}$$

Since $0 < \rho(\lambda_{t-1}) < 1$, both beliefs converge to the prior as $k \to \infty$:

$$\lim_{k\to\infty} \bar{\lambda}^H_{t,k} = \lim_{k\to\infty} \bar{\lambda}^L_{t,k} = \lambda_{t-1}.$$

Higher-order beliefs progressively lose information about the realized regime. Stronger strategic complementarity, which places greater weight on higher-order beliefs in (9), consequently makes prices less responsive to the regime actually chosen.

Finally, since $g^L = 0$, the kth-order average expectation of money growth under regime K is

$$g^{(k)}_{t,K} = g^H\left(1 - \bar{\lambda}^K_{t,k}\right), \qquad K \in \{H, L\}.$$

D Proofs for the Analytical Model

D.1 Proof of Proposition 1

Proposition 1.
Fix $\lambda_0 \in (0,1)$ and suppose private signals are noisy and informative. Under imperfect common knowledge in period 1,

$$y^L_1(\lambda_0) < 0 < y^H_1(\lambda_0), \qquad 0 < \pi^L_1(\lambda_0) < \pi^H_1(\lambda_0) < g^H.$$

If the regime is common knowledge in period 1, then $y^K_1 = 0$ and $\pi^K_1 = g^K$.

Proof. Let $g^{(k)}_{1,K}(\lambda_0)$ denote the kth-order average expectation of the period-1 money growth rate conditional on the realized regime $K \in \{H, L\}$. Since $g^L = 0$,

$$g^{(k)}_{1,K}(\lambda_0) = g^H\left(1 - \bar{\lambda}^K_{1,k}\right).$$

Equation (9) therefore implies

$$\pi_1^K(\lambda_0) = g^H \left[1 - \sum_{k=1}^{\infty} \xi(1-\xi)^{k-1} \bar{\lambda}_{1,k}^K\right].$$

Using (32),

$$\sum_{k=1}^{\infty} \xi(1-\xi)^{k-1} \bar{\lambda}_{1,k}^H = \frac{\alpha(\lambda_0)}{1-(1-\xi)\rho(\lambda_0)}$$

and

$$\sum_{k=1}^{\infty} \xi(1-\xi)^{k-1} \bar{\lambda}_{1,k}^L = \frac{\alpha(\lambda_0)+\xi\rho(\lambda_0)}{1-(1-\xi)\rho(\lambda_0)}.$$

It follows that

$$y_1^H(\lambda_0) = g^H \frac{\alpha(\lambda_0)}{1-(1-\xi)\rho(\lambda_0)}$$

and

$$y_1^L(\lambda_0) = -g^H \frac{1-\beta(\lambda_0)}{1-(1-\xi)\rho(\lambda_0)}.$$

Noisy informative signals imply $0 < \alpha(\lambda_0) < \beta(\lambda_0) < 1$, and $0 < \xi < 1$. The denominator is therefore positive. Since $g^H > 0$, these expressions imply

$$y_1^L(\lambda_0) < 0 < y_1^H(\lambda_0).$$

The quantity equation gives $\pi_1^H = g^H - y_1^H$ and $\pi_1^L = -y_1^L$. Subtracting the two inflation rates gives

$$\pi_1^H - \pi_1^L = g^H \left[1 - \frac{1-\rho(\lambda_0)}{1-(1-\xi)\rho(\lambda_0)}\right] = g^H \frac{\xi\rho(\lambda_0)}{1-(1-\xi)\rho(\lambda_0)} > 0.$$

Together with the output inequalities, this yields $0 < \pi_1^L < \pi_1^H < g^H$.

Under common knowledge in period 1, $g_{1,K}^{(k)} = g^K$ at every order. Because the weights in (9) sum to one, $\pi_1^K = g^K$, so $y_1^K = 0$. □

D.2 Proof of Proposition 2

Proposition 2.

Fix $\lambda_0 \in (0,1)$ under imperfect common knowledge in period 1. The monetary authority chooses g^L if and only if

$$\theta \leq T(\lambda_0).$$

If $T(\lambda_0) < \bar{\theta}$, every type in the support with $\theta > T(\lambda_0)$ chooses g^H. Under common knowledge in period 1, every type chooses g^L.

Proof. Under imperfect common knowledge, the monetary authority chooses g^L if and only if

$$-\theta y_1^L(\lambda_0) + [\pi_1^L(\lambda_0)]^2 \leq -\theta y_1^H(\lambda_0) + [\pi_1^H(\lambda_0)]^2.$$

Rearranging and using $y_1^H(\lambda_0) - y_1^L(\lambda_0) > 0$, this condition is equivalent to

$$\theta \leq \frac{[\pi_1^H(\lambda_0)]^2 - [\pi_1^L(\lambda_0)]^2}{y_1^H(\lambda_0) - y_1^L(\lambda_0)} = T(\lambda_0).$$

Every type $\theta > T(\lambda_0)$ therefore chooses g^H.

Under common knowledge in period 1, Proposition 1 shows that both output gaps are zero, $\pi_1^L = 0$, and $\pi_1^H = g^H > 0$. Hence

$$\mathcal{L}_1(g^L; \theta) = 0 < (g^H)^2 = \mathcal{L}_1(g^H; \theta)$$

for every type, so every type chooses g^L. □

D.3 Proof of Proposition 3

Proposition 3.
Suppose firms set prices in URV units. Under either regime $g_1 \in \{g^L, g^H\}$ the unique equilibrium satisfies

$$x_1(i) = 0, \qquad p_1^U = 0, \qquad y_1 = 0, \qquad \pi_1 = g^K.$$

Inflation is zero under g^L and equals g^H under g^H. Every monetary-authority type chooses g^L.

Proof. Under (13), $u_1 = g_1$, so the money growth rate in URV units is $g_1^U = 0$ under either regime. Every order of average beliefs about g_1^U is therefore also zero, and (15) gives

$$p_1^U = 0 \qquad \text{and} \qquad y_1 = 0.$$

The same conclusion follows directly from the pricing recursion. With $g_1^U = 0$, (14) implies $x_1(i) = (1-\xi)\mathbb{E}_1^i[p_1^U]$. By iterating the aggregate condition, we get a bounded remainder multiplied by $(1-\xi)^n$, which converges to zero because $0 < \xi < 1$. Thus $p_1^U = 0$ is the unique bounded solution, and every firm chooses $x_1(i) = 0$.

Since $u_1 = g_1$ and $p_1^U = 0$, inflation satisfies $\pi_1 = u_1 + p_1^U = g_1$. The monetary authority's loss is zero under g^L and $(g^H)^2 > 0$ under g^H, so every type chooses stabilization. □

D.4 Proof of Proposition 4

Proposition 4 (Endpoint credibility).
Suppose $0 < \underline{\theta} < \bar{\theta}$. *Under domestic-currency pricing,* $\lambda_0^* = 0$ *is always an equilibrium, while* $\lambda_0^* = 1$ *is never an equilibrium.*

Proof. At $\lambda_0 = 0$, firms assign probability one to g^H after every signal and set prices implying $\pi_1 = g^H$ under either policy choice. If the authority chooses g^H, then

$$y_1^H = 0, \qquad \pi_1^H = g^H.$$

If it instead chooses g^L, then

$$y_1^L = -g^H, \qquad \pi_1^L = g^H.$$

For every $\theta > 0$,

$$\mathcal{L}_1(g^L; \theta) = \theta g^H + (g^H)^2 > (g^H)^2 = \mathcal{L}_1(g^H; \theta).$$

Every authority type chooses g^H, so the fraction choosing stabilization is zero and $\lambda_0^* = 0$ is an equilibrium.

At $\lambda_0 = 1$, firms assign probability one to g^L after every signal and set prices implying $\pi_1 = 0$ under either policy choice. If the authority chooses g^L, then

$$y_1^L = 0, \qquad \pi_1^L = 0.$$

If it instead chooses g^H, then

$$y_1^H = g^H, \qquad \pi_1^H = 0.$$

For every $\theta > 0$,

$$\mathcal{L}_1(g^H; \theta) = -\theta g^H < 0 = \mathcal{L}_1(g^L; \theta).$$

Every authority type chooses g^H, so the fraction choosing stabilization is zero rather than one. Hence $\lambda_0^* = 1$ cannot be an equilibrium. □

D.5 Proof of Proposition 5

Proposition 5.
Suppose F_θ is continuous with support $0 < \underline{\theta} < \bar{\theta}$. Under domestic-currency pricing, any equilibrium other than $\lambda_0^* = 0$ is interior and satisfies

$$\lambda_0^* = F_\theta\left(T(\lambda_0^*)\right), \qquad 0 < \lambda_0^* < 1.$$

Under the URV, the unique equilibrium has $\lambda_0^* = 1$. Every monetary authority type chooses g^L, and $y_1 = \pi_1 = 0$.

Proof. Proposition 4 establishes that $\lambda_0^* = 0$ is an equilibrium and $\lambda_0^* = 1$ is not. Hence every equilibrium other than $\lambda_0^* = 0$ must be interior. For an interior prior λ_0, Proposition 2 implies that precisely the authority types with $\theta \leq T(\lambda_0)$ choose g^L. The fraction of types choosing stabilization is therefore $F_\theta(T(\lambda_0))$, so consistency with firms' prior requires

$$\lambda_0^* = F_\theta\left(T(\lambda_0^*)\right).$$

Conversely, any interior solution to this equation, together with the cutoff strategy, Bayesian posteriors, and the pricing outcome in Proposition 1, satisfies the equilibrium conditions.

Under the URV, Proposition 3 implies that every authority type chooses g^L, independently of firms' prior. Consistency therefore requires $\lambda_0^* = 1$, and the same proposition gives $y_1 = \pi_1 = 0$. □

D.6 Proof of Proposition 6

Proposition 6 (Uniform authority types).
Suppose $\theta \sim U[0, \bar{\theta}]$, with $\bar{\theta} > 0$. Define

$$\kappa \equiv 2g^H \xi \left[\exp\left(\frac{(g^H)^2}{\sigma_g^2}\right) - 1\right].$$

Under domestic-currency pricing, $\lambda_0^* = 0$ is always an equilibrium. If $\bar{\theta} \geq \kappa$, it is the unique equilibrium. If $0 < \bar{\theta} < \kappa$, there is also exactly one interior equilibrium $\lambda_0^I \in (0, 1)$, characterized by

$$T(\lambda_0^I) = \bar{\theta}\lambda_0^I, \qquad \theta^I = \bar{\theta}\lambda_0^I.$$

Under the URV, the unique equilibrium has $\lambda_0^* = 1$.

Proof. Let f_H and f_L denote the Gaussian signal densities in Appendix C.2. For $0 < \lambda_0 < 1$, define

$$g_{\lambda_0}(z) = (1-\lambda_0) f_H(z) + \lambda_0 f_L(z), \qquad J(\lambda_0) = \int \frac{f_H(z) f_L(z)}{g_{\lambda_0}(z)}\, dz.$$

Using (30) and the definition of ρ as the difference in average posterior probabilities across regimes gives

$$\rho(\lambda_0) = \int \lambda_1(z;\lambda_0)\big[f_L(z) - f_H(z)\big]\, dz = 1 - J(\lambda_0).$$

In particular, $J(0) = J(1) = 1$, and $0 < J(\lambda_0) < 1$ for an interior prior.

Define the transformation

$$s(\lambda_0) \equiv \frac{\xi \rho(\lambda_0)}{1-\rho(\lambda_0)} = \xi\left[\frac{1}{J(\lambda_0)} - 1\right].$$

It satisfies $s(0) = s(1) = 0$ and $s(\lambda_0) > 0$ for $0 < \lambda_0 < 1$. Substituting the outcomes $y_1^K(\lambda_0)$ and $\pi_1^K(\lambda_0)$ from the proof of Proposition 1, together with $\alpha(\lambda_0) = (1-\rho(\lambda_0))\lambda_0$ and $1-\beta(\lambda_0) = (1-\rho(\lambda_0))(1-\lambda_0)$, into the definition of $T(\lambda_0)$ gives

$$\frac{T(\lambda_0)}{g^H} = \frac{s(\lambda_0)\big[2(1-\lambda_0) + s(\lambda_0)\big]}{1+s(\lambda_0)} = s(\lambda_0) + (1-2\lambda_0)\frac{s(\lambda_0)}{1+s(\lambda_0)}. \tag{33}$$

I first establish the shape of s. Differentiating under the integral sign gives

$$J'(\lambda_0) = -\int \frac{f_H(z) f_L(z)\big[f_L(z) - f_H(z)\big]}{g_{\lambda_0}(z)^2}\, dz,$$
$$J''(\lambda_0) = 2\int \frac{f_H(z) f_L(z)\big[f_L(z) - f_H(z)\big]^2}{g_{\lambda_0}(z)^3}\, dz.$$

Cauchy–Schwarz, applied with weight $f_H f_L / g_{\lambda_0}$, implies

$$[J'(\lambda_0)]^2 < \frac{1}{2} J(\lambda_0) J''(\lambda_0).$$

The inequality is strict because $(f_L - f_H)/g_{\lambda_0}$ is nonconstant. Therefore

$$s''(\lambda_0) = \xi\, \frac{2[J'(\lambda_0)]^2 - J(\lambda_0)J''(\lambda_0)}{J(\lambda_0)^3} < 0.$$

Because the Gaussian densities have the same variance and $g^L = 0$, replacing z by $g^H - z$ interchanges f_H and f_L. Hence $J(\lambda_0) = J(1-\lambda_0)$ and $s(\lambda_0) = s(1-\lambda_0)$. Strict concavity

and symmetry imply

$$s'(\lambda_0) > 0 \quad \left(0 < \lambda_0 < \frac{1}{2}\right), \qquad s'\left(\frac{1}{2}\right) = 0, \qquad s'(\lambda_0) < 0 \quad \left(\frac{1}{2} < \lambda_0 < 1\right).$$

Next, differentiating (33) and suppressing the argument of s gives

$$\frac{T'(\lambda_0)}{g^H} = \left[1 + \frac{1 - 2\lambda_0}{(1+s)^2}\right] s' - \frac{2s}{1+s}, \tag{34}$$

and

$$\frac{T''(\lambda_0)}{g^H} = \left[1 + \frac{1 - 2\lambda_0}{(1+s)^2}\right] s'' - \frac{4s'}{(1+s)^2} - \frac{2(1-2\lambda_0)(s')^2}{(1+s)^3}. \tag{35}$$

For $0 < \lambda_0 < 1/2$, every term in (35) is negative, so $T''(\lambda_0) < 0$. For $1/2 \leq \lambda_0 < 1$, the coefficient on s' in (34) is positive, $s' \leq 0$, and $s > 0$. Thus

$$T''(\lambda_0) < 0 \quad \left(0 < \lambda_0 < \frac{1}{2}\right), \qquad T'(\lambda_0) < 0 \quad \left(\frac{1}{2} \leq \lambda_0 < 1\right).$$

These properties imply that $T(\lambda_0)/\lambda_0$ is strictly decreasing on $(0, 1)$. On the first half of the interval, strict concavity and $T(0) = 0$ imply $T(\lambda_0) > \lambda_0 T'(\lambda_0)$. On the second half, the same inequality follows from $T(\lambda_0) > 0$ and $T'(\lambda_0) < 0$. Consequently,

$$\frac{d}{d\lambda_0}\left[\frac{T(\lambda_0)}{\lambda_0}\right] = \frac{\lambda_0 T'(\lambda_0) - T(\lambda_0)}{\lambda_0^2} < 0. \tag{36}$$

It remains to calculate the limit at zero. The Gaussian densities allow differentiation from the right at the endpoint, giving

$$J'(0^+) = 1 - \int \frac{f_L(z)^2}{f_H(z)}\, dz.$$

Completing the square yields

$$\frac{f_L(z)^2}{f_H(z)} = \exp\left(\frac{(g^H)^2}{\sigma_g^2}\right) \frac{1}{\sigma_g} \phi\left(\frac{z + g^H}{\sigma_g}\right),$$

so

$$s'(0^+) = -\xi J'(0^+) = \xi \left[\exp\left(\frac{(g^H)^2}{\sigma_g^2}\right) - 1\right].$$

Since $s(0) = 0$, equation (34) implies $T'(0^+) = 2g^H s'(0^+) = \kappa$. Therefore

$$\lim_{\lambda_0 \downarrow 0} \frac{T(\lambda_0)}{\lambda_0} = \kappa, \qquad \lim_{\lambda_0 \uparrow 1} \frac{T(\lambda_0)}{\lambda_0} = 0. \tag{37}$$

For $\theta \sim U[0, \bar{\theta}]$, an interior equilibrium must satisfy

$$\lambda_0 = \frac{T(\lambda_0)}{\bar{\theta}}, \qquad \text{or equivalently} \qquad \frac{T(\lambda_0)}{\lambda_0} = \bar{\theta}.$$

Conversely, any solution in $(0, 1)$ has $T(\lambda_0) = \bar{\theta}\lambda_0 \in (0, \bar{\theta})$, so it satisfies the uniform-distribution fixed point. By (36) and (37), there is exactly one such solution when $0 < \bar{\theta} < \kappa$, and none when $\bar{\theta} \geq \kappa$. In particular, equality does not support an interior equilibrium because $T(\lambda_0)/\lambda_0 < \kappa$ at every positive prior.

It remains to check the endpoint priors. When $\lambda_0 = 0$, every firm is certain of g^H, so $\pi_1 = g^H$ under either regime. Choosing g^H gives $y_1 = 0$ and loss $(g^H)^2$, while choosing g^L gives $y_1 = -g^H$ and loss $\theta g^H + (g^H)^2$. Every type with $\theta > 0$ therefore strictly prefers g^H. Only $\theta = 0$ is indifferent, and it has zero mass under $U[0, \bar{\theta}]$. The fraction of types choosing g^L is zero, so $\lambda_0^* = 0$ is an equilibrium. When $\lambda_0 = 1$, every firm is certain of g^L, so $\pi_1 = 0$ under either regime. Choosing g^L gives $y_1 = 0$ and loss 0, while choosing g^H gives $y_1 = g^H$ and loss $-\theta g^H$. Every type with $\theta > 0$ strictly prefers g^H, so the fraction of types choosing g^L is again zero and $\lambda_0^* = 1$ is not an equilibrium. The cutoff at the interior equilibrium is $\theta^I = T(\lambda_0^I) = \bar{\theta}\lambda_0^I$.

Under the URV, $u_1 = g_1$, so money growth in URV units is zero under either regime, and the unique outcome in URV units has $x_1(i) = 0$ for every firm. Hence $y_1 = 0$ and $\pi_1 = g_1$ for any prior. The loss is 0 under g^L and $(g^H)^2 > 0$ under g^H, so every type, including $\theta = 0$, chooses g^L. Consistency with firms' prior then requires $\lambda_0^* = 1$, which is the unique equilibrium. □

E Extensions

This appendix extends the analytical model of Section 3 in two directions. The first lets disinflation raise the demand for money. The second adds contracts indexed to past inflation. Both cases increase the cost of a disinflation. The URV successfully avoids the costs in both cases.

E.1 Varying Money Velocity

It is a well-known stylized fact that following the end of hyperinflation, money velocity decreases. I show that dropping the constant money velocity assumption from my analytical model increases the recession and makes the choice of not using the URV even more costly.

The end of hyperinflation is usually followed by an increase in real money balances (Sargent, 1982; Dornbusch and Simonsen, 1987). For example, in Brazil, M0 increased by 370% in real terms in the six months following the Real introduction. This represents a reduction in money velocity of 76% in that period. It may seem counterintuitive that the money base would grow so much right when hyperinflation ends. The intuition, however, is simple. As the inflation rate falls, the opportunity cost of holding cash lowers. Households, then, demand more cash, which the monetary authority accommodates, thereby lowering money velocity.

The increase in demand for real money balances following the end of hyperinflation is known as remonetization. Remonetization has two important consequences for stabilization. First, because households are increasing their real money balance, the government can expand the nominal monetary base without generating inflation. This provides a temporary source of revenue, which may offset the loss of seigniorage revenue following stabilization. This additional revenue disappears once the demand for real money balances stabilizes. However, a permanently higher money supply can continue to reduce the government's interest burden since cash pays zero interest.

Second, remonetization creates a signaling issue for the monetary authority. If it keeps the nominal monetary base fixed despite the increase in money demand, an equilibrium in the money market can only be reached through a decline in the price level, which increases the real money supply. With rigidities, this adjustment may generate a recession. The central bank can avoid this contraction by expanding the monetary base to accommodate higher money demand. This expansion, however, may undermine the credibility of stabilization if agents interpret it as a return to the high money growth rate regime. The monetary authority then faces a trade-off between accommodating remonetization and signaling a low money growth rate regime. The 1990 Brazilian Collor Plan is a great example of how catastrophic this can be.

In the baseline model, I assume that money velocity is constant. I now allow disinflation to raise real money demand, which lowers money velocity. Let $V_0 > 0$ denote initial velocity, with $M_0 V_0 = P_0 \bar{Y}$, and take the inherited inflation rate to be g^H. I use Cagan's money demand function, using the effective inflation rate instead of the expected inflation rate. I

replace constant velocity with the money demand relation

$$\frac{M_t^d}{P_t} = \frac{Y_t}{V_{t-1}} \exp\left[\psi(g^K - \pi_t)\right], \qquad 0 \leq \psi < 1. \tag{38}$$

Where $K \in \{H, L\}$ denotes the inherited money growth rate regime, and ψ measures the response of real money demand per unit of output to a fall in inflation. Everything else stays the same as in Section 3. The case $\psi = 0$ recovers the baseline, while $\psi < 1$ ensures that the pricing recursion below remains a contraction.

This addition does not change the equilibrium and threshold results from Propositions 2, 4, and 5. The main difference comes from studying the consequences for output when money velocity is not constant.

Money market clearing, $M_t = M_t^d$, implies

$$\log\left(\frac{V_t}{V_{t-1}}\right) = \psi(\pi_t - g^H), \qquad y_t = g_t - \psi g^H - (1-\psi)\pi_t. \tag{39}$$

When inflation falls below its inherited rate, desired real balances per unit of output rise and velocity falls. Under $g^L = 0$, the nominal money stock remains fixed, and the output gap is $y_t = -\pi_t - \psi(g^H - \pi_t)$. Relative to the baseline quantity equation, the fall in velocity reduces aggregate expenditure further at any given inflation rate below g^H.

Substituting (39) into (4) gives

$$p_t(i) = \xi\left(\mathbb{E}_t^i[g_t] - \psi g^H\right) + \left[1 - \xi(1-\psi)\right]\mathbb{E}_t^i[\pi_t]. \tag{40}$$

A larger ψ raises the coefficient on expected aggregate inflation. If other firms lower their prices, inflation falls, and money demand per unit of output rises, reducing nominal expenditure for a given money supply. This, in turn, gives the firms more incentives to lower their own prices. As a result, the interaction between money demand and pricing increases the complementarity in firms' decisions.

Proposition 7 *Fix $\lambda_0 \in (0,1)$ and suppose private signals are noisy and informative. Under (38), the domestic-currency pricing equilibrium is unique. Holding the prior and signal structure fixed, both output gaps increase in absolute value with ψ. In particular, for $0 < \psi < 1$,*

$$y_{1,\psi}^L < y_{1,0}^L < 0 < y_{1,0}^H < y_{1,\psi}^H.$$

Under common knowledge of the realized regime, output is at its natural level under both regimes. For $\psi > 0$, stabilization with a fixed nominal money stock requires deflation.

Proof. See Appendix F.1.

Proposition 7 shows that remonetization reinforces the output effects of incomplete common knowledge. As ψ rises, the pricing recursion places more weight on higher-order beliefs. The recession under g^L becomes deeper, while the expansion under g^H becomes larger. Under common knowledge, prices adjust enough to satisfy the higher demand for real balances without reducing output.

I next consider the URV. Because disinflation raises money demand, I allow the monetary authority to make a one-time level adjustment to the money stock on top of its regime choice. This is meant to allow the monetary authority to accommodate remonetization in the period right after the end of hyperinflation:[16]

$$g_1^{\text{tot}} \equiv \log\left(\frac{M_1}{M_0}\right) = g_1 + a_1, \qquad g_1 \in \{g^L, g^H\}, \qquad a_1 \geq 0,$$

where g_1 is the regime component and a_1 the one-time level adjustment. The URV follows $u_1 = g_1$, so the conversion factor tracks only the regime component. Money market clearing in the URV economy gives

$$\begin{aligned} y_1 &= g_1 + a_1 - \psi g^H - (1-\psi)(u_1 + p_1^U) \\ &= a_1 - \psi(g^H - g_1) - (1-\psi)p_1^U. \end{aligned} \tag{41}$$

Proposition 8 compares output under the baseline URV rule, which sets $a_1 = 0$, with output when the monetary authority accommodates remonetization through a one-time increase in the monetary base. Under this accommodation rule,

$$a_1 = \psi(g^H - g_1), \tag{42}$$

the authority increases the monetary base by $a_1 = \psi(g^H - g^L)$ when $g_1 = g^L$, and makes no adjustment when $g_1 = g^H$. Both rules are common knowledge. The money supply increase and the corresponding adjustment to the conversion factor are set after firms choose their prices, so the realized monetary regime remains private information at the time of price setting.

An important advantage of the URV is that it allows the monetary authority to accommodate remonetization without undermining nominal price coordination. With the URV, the credibility of the stabilization plan depends only on the conversion rule rather than on the money growth rate. As long as the URV conversion rule remains credible, a one-time

[16] Without the URV, if this adjustment did not affect firms' expectations, the model with varying velocity would deliver exactly the same results as Section 3.

monetary expansion does not affect price coordination. Without the URV, stabilization relies directly on firms' expectations of future monetary growth. A large increase in the monetary base could undermine stabilization if firms interpret it as a return to the high money growth rate regime.

Proposition 8 *Suppose firms set prices in URV. Then,*

(a) Under the baseline rule $a_1 = 0$ and $\lambda_0 \in (0,1)$, the unique equilibrium satisfies

$$y_{1,\psi}^{K,U} = \psi\, y_{1,\psi}^{K}, \qquad K \in \{H, L\},$$

so that $y_{1,\psi}^{L} < y_{1,\psi}^{L,U} < 0 < y_{1,\psi}^{H,U} < y_{1,\psi}^{H}$.

(b) Under the accommodation rule, the unique equilibrium satisfies $x_1(i) = p_1^U = y_1 = 0$ and $\pi_1 = g^K$. Every monetary authority type chooses g^L, and the unique equilibrium has $\lambda_0^* = 1$, $\pi_1 = y_1 = 0$.

Proof. See Appendix F.2.

Part (a) shows that the baseline conversion rule reduces the output cost of incomplete common knowledge but no longer eliminates it. With $a_1 = 0$, the component of expenditure left for firms to forecast is $\psi(g_1 - g^H)$. It is negative under g^L and zero under g^H, so uncertainty about the regime still affects URV prices, and the output gap is a fraction ψ of the gap under domestic-currency pricing. Part (b) shows that a one-time monetary expansion closes this gap. Under (42), output satisfies $y_1 = -(1-\psi)p_1^U$, which removes the regime from firms' real-price problem. As a result, firms know that the monetary authority cannot affect their decisions and so will always choose to stabilize. Stabilization is the only equilibrium with $\pi_1 = y_1 = 0$.

E.2 Lagged Indexation

Widespread indexation is a distinctive feature of long periods of high inflation (Pazos, 1972). In this section, I analyze the effects of lagged indexation in the analytical model. The results are similar to those with a varying money velocity, in the sense that lagged indexation increases the output cost of a money-based disinflation, while the URV can make it zero.

I start from the baseline model in Section 3 and assume that a few have their prices indexed to last period's inflation rate. Initially, all firms charge $P_0(i) = P_0 = 1$, output is at its natural level, and the inherited inflation rate is $\pi_0 = g^H$. A fraction $\omega \in [0,1]$ of firms indexes their price by

$$P_1(i) = P_0(i)e^{\pi_0}, \qquad p_1(i) = \pi_0 = g^H. \tag{43}$$

These firms cannot revise their indexation clause or reset the contract price in period 1. The idea is that these firms already signed contracts for the next period and so were caught off guard by the monetary authority's decision to lower the money growth rate. The remaining fraction $1-\omega$ chooses prices freely using (5). I study inherited contracts that expire before period 2, when the regime becomes common knowledge, so the analysis concerns only the impact cost of stabilization.

As with the model without constant money velocity, lagged indexation does not change the equilibrium and threshold results from Propositions 2, 4, and 5. The main difference comes from studying the consequences for output when we have lagged indexation.

The aggregate price index combines inherited inflation with the prices chosen by optimizing firms. Averaging the two pricing rules gives

$$\pi_1 = \omega g^H + (1-\omega)\left[\xi g_1^{(1)} + (1-\xi)\pi_1^{(1)}\right]. \tag{44}$$

Proposition 9 *With lagged indexation, both output gaps are strictly decreasing in ω. In particular, for $0<\omega<1$,*

$$y_{1,\omega}^L < y_{1,0}^L < 0 < y_{1,\omega}^H < y_{1,0}^H.$$

At $\omega = 1$, $\pi_{1,1}^H = \pi_{1,1}^L = g^H$, $y_{1,1}^H = 0$, and $y_{1,1}^L = -g^H$. Under common knowledge of the realized regime, stabilization still produces positive inflation and a negative output gap whenever $\omega > 0$, while continued high money growth leaves output at its natural level and inflation at g^H.

Proof. See Appendix F.3.

Proposition 9 establishes two important results. First, lagged indexation deepens the recession following stabilization but reduces the boom when high money growth persists. The intuition is that indexation makes inflation less responsive to changes in the money growth rate. When the monetary authority reduces money growth to zero, firms with indexed prices still see their prices growing at the previous period's high inflation rate. As a result, aggregate inflation falls by less than money growth, leading to a larger output loss. When the monetary authority maintains a high money growth regime, indexation dampens the output boom. Without indexation, firms restrain their price increases because they expect other firms to anticipate stabilization. With indexation, however, some firms automatically raise their prices by the previous period's inflation rate. Optimizing firms anticipate this, and even those that expect stabilization end up raising their prices, reducing the monetary authority's ability to fool firms into producing an output boom.

The second result is that when some firms are indexing their prices by lagged inflation, a

disinflation is costly even under complete common knowledge. The reason is that firms with indexed contracts continue raising their prices at the previous period's inflation rate, regardless of their beliefs. Optimizing firms anticipate this and increase their prices because of strategic complementarities. As a result, inflation does not adjust to the reduction in money growth, generating a recession even when all firms know that stabilization is common knowledge. This result highlights an important distinction between informational and contractual frictions: eliminating uncertainty about monetary policy is not enough to achieve costless stabilization when prices remain indexed to past inflation. Breaking backward-looking indexation is crucial for reducing the output costs of disinflation.

It is precisely because of the combination of informational and contractual problems that makes the URV a great policy tool. The URV works not only by shutting down the informational friction. It also works by removing backward-looking indexation. By providing firms with a contemporaneous indexation instrument, firms can now stop using past inflation to adjust their prices, and can instead use current inflation. In Brazil, during the URV period, indexation was forbidden. Agents could only use the URV to index prices and wages. As we show below, this feature eliminates the output cost associated with a disinflation.

I now formalize the role of the URV in this economy. Optimizing firms choose prices in URV, with $u_t = g_t$. Since $g_t - u_t = 0$, their pricing condition is $x_t(i) = (1-\xi)\mathbb{E}_t^i[p_t^U]$, and output satisfies $y_t = -p_t^U$.

I compare two URV options for the inherited contracts. In the first, I analyze the case where the URV does nothing to indexed contracts. That is, contracts retain their nominal indexation clause (43). An indexed firm's price measured in URV is then $x_t(i) = g^H - g_t$, rather than a price it can choose before the conversion factor is applied, and the aggregate price level in URV satisfies

$$p_t^U = \omega(g^H - g_t) + (1-\omega)(1-\xi)(p_t^U)^{(1)}, \tag{45}$$

Notice that the inherited nominal contracts leave a regime-dependent component in the aggregate price level measured in URV. This means that the monetary authority does not have full control of the prices chosen by firms today, which will generate an output loss following a decrease in the money growth rate.

In the second case, I follow an approach closer to the Real Plan, in which indexed contracts were also converted into URV:

$$P_1(i) = P_0(i)U_1, \tag{46}$$

This conversion replaces (43). In both cases, the rule is common knowledge, and $u_1 = g_1$ is applied after optimizing firms choose prices.

Proposition 10 *Suppose optimizing firms set prices in URV with $u_1 = g_1$. Then,*

(a) If inherited contracts retain their nominal lagged-indexation clause, $\lambda_0 \in (0,1)$, and $0 < \omega < 1$, the unique equilibrium satisfies

$$y^L_{1,\omega} < y^{L,U}_{1,\omega} < 0, \qquad y^{H,U}_{1,\omega} < 0 < y^H_{1,\omega}.$$

Under common knowledge of the realized regime, the output gaps coincide with those under domestic-currency pricing.

(b) If inherited contracts are converted according to (46), then for any $0 \leq \omega \leq 1$ and any prior, the unique equilibrium satisfies $x_1(i) = p^U_1 = y_1 = 0$ and $\pi_1 = g^K$. Every monetary authority type chooses g^L, and the unique equilibrium has $\lambda^*_0 = 1$ and $\pi_1 = y_1 = 0$.

Proof. See Appendix F.4.

Part (a) shows that changing the unit in which optimizing firms set prices reduces the recession under stabilization but does not eliminate it. It can only remove the cost of incomplete common knowledge, but it cannot avoid the output cost coming from lagged indexation. Under g^L, the indexed contracts still increase even though the URV conversion factor is constant, which makes firms setting prices in URV have to increase their price. Under g^H, the contracts are consistent with natural output, but optimizing firms assign positive probability to stabilization, under which indexed prices would be high relative to the URV. They therefore choose positive URV prices, generating a contraction even when high money growth continues.

Part (b) shows that converting the indexed prices into URV removes both costs. The conversion is relatively simple; it just replaces the lagged indexation with the URV conversion factor, so the inherited inflation component never enters firms' real-price decisions. Indexed firms keep their initial URV prices, and optimizing firms face no regime-dependent component of nominal expenditure in URV units. The monetary authority's loss is then zero under stabilization and $(g^H)^2$ under continued high money growth, so every type chooses stabilization.

F Proofs for the Extensions

F.1 Proof of Proposition 7

Proposition 7.
Fix $\lambda_0 \in (0,1)$ and suppose private signals are noisy and informative. Under (38), the domestic-currency pricing equilibrium is unique. Holding the prior and signal structure fixed, both output gaps increase in absolute value with ψ. In particular, for $0 < \psi < 1$,

$$y^L_{1,\psi} < y^L_{1,0} < 0 < y^H_{1,0} < y^H_{1,\psi}.$$

Under common knowledge of the realized regime, output is at its natural level under both regimes. For $\psi > 0$, stabilization with a fixed nominal money stock requires deflation.

Proof. Define $\widetilde{\xi} \equiv \xi(1-\psi)$ and

$$D_\psi(\lambda_0) \equiv 1 - (1-\widetilde{\xi})\rho(\lambda_0),$$

where $\alpha(\lambda_0)$, $\beta(\lambda_0)$, and $\rho(\lambda_0)$ are the posterior moments defined in Appendix C.3. These moments depend on the prior and signal structure, which I hold fixed when varying ψ. Let $y^K_{1,\psi}(\lambda_0)$ and $\pi^K_{1,\psi}(\lambda_0)$ denote the domestic-currency outcome under regime $K \in \{H, L\}$.

Averaging (40) gives

$$\pi_1 = \xi\left(g_1^{(1)} - \psi g^H\right) + (1-\widetilde{\xi})\pi_1^{(1)}.$$

Since $0 < \widetilde{\xi} < 1$, iteration leaves a bounded remainder multiplied by $(1-\widetilde{\xi})^n$, which converges to zero. Define

$$\mathcal{G}_{1,\psi} \equiv \sum_{k=1}^{\infty} \widetilde{\xi}(1-\widetilde{\xi})^{k-1} g_1^{(k)}.$$

The unique bounded aggregate solution is

$$\pi_1 = \frac{\mathcal{G}_{1,\psi} - \psi g^H}{1-\psi}, \qquad y_1 = g_1 - \mathcal{G}_{1,\psi}. \tag{47}$$

The pricing rule then determines each firm's price uniquely.

Using (32) and summing the geometric series gives

$$\sum_{k=1}^{\infty} \widetilde{\xi}(1-\widetilde{\xi})^{k-1}\bar{\lambda}_{1,k}^{H} = \frac{\alpha}{D_\psi},$$
$$\sum_{k=1}^{\infty} \widetilde{\xi}(1-\widetilde{\xi})^{k-1}\bar{\lambda}_{1,k}^{L} = \frac{\alpha+\widetilde{\xi}\rho}{D_\psi},$$

Since $g_{1,K}^{(k)} = g^H(1-\bar{\lambda}_{1,k}^{K})$, equation (47) implies

$$y_{1,\psi}^{H} = g^H\frac{\alpha}{D_\psi}, \qquad y_{1,\psi}^{L} = -g^H\frac{1-\beta}{D_\psi}.$$

Here $D_\psi = 1-\rho+\xi(1-\psi)\rho > 0$, and noisy informative signals imply $0<\alpha<\beta<1$. The output gaps have the stated signs. Holding the prior and signals fixed,

$$\frac{\partial y_{1,\psi}^{H}}{\partial\psi} = \frac{g^H\alpha\xi\rho}{D_\psi^2} > 0, \qquad \frac{\partial y_{1,\psi}^{L}}{\partial\psi} = -\frac{g^H(1-\beta)\xi\rho}{D_\psi^2} < 0.$$

At $\psi = 0$, these expressions recover the outcomes in Proposition 1, establishing the strict comparisons with the baseline. Inflation follows from (39):

$$\pi_{1,\psi}^{K}(\lambda_0) = \frac{g^K - \psi g^H - y_{1,\psi}^{K}(\lambda_0)}{1-\psi}.$$

Under common knowledge of the realized regime, $g_1^{(k)} = g^K$ at every order. The weights defining $\mathcal{G}_{1,\psi}$ sum to one, so $\mathcal{G}_{1,\psi} = g^K$. Equation (47) then gives $y_{1,\psi}^{K} = 0$ and $\pi_{1,\psi}^{K} = (g^K - \psi g^H)/(1-\psi)$. □

F.2 Proof of Proposition 8

Proposition 8.
Suppose firms set prices in URV. Then,

(a) Under the baseline rule $a_1 = 0$ and $\lambda_0 \in (0,1)$, the unique equilibrium satisfies

$$y_{1,\psi}^{K,U} = \psi\, y_{1,\psi}^{K}, \qquad K \in \{H, L\},$$

so that $y_{1,\psi}^{L} < y_{1,\psi}^{L,U} < 0 < y_{1,\psi}^{H,U} < y_{1,\psi}^{H}$.

(b) Under the accommodation rule, the unique equilibrium satisfies $x_1(i) = p_1^U = y_1 = 0$

and $\pi_1 = g^K$. Every monetary authority type chooses g^L, and the unique equilibrium has $\lambda_0^* = 1$, $\pi_1 = y_1 = 0$.

Proof. For a price chosen in URV, the log-linearized optimality condition is

$$x_1(i) = \mathbb{E}_1^i[p_1^U] + \xi \mathbb{E}_1^i[y_1],$$

and output satisfies (41).

Part (a). With $a_1 = 0$, equation (41) becomes

$$y_1 = \psi(g_1 - g^H) - (1-\psi)p_1^U.$$

Substituting into the pricing condition gives

$$x_1(i) = \xi\psi\left(\mathbb{E}_1^i[g_1] - g^H\right) + (1-\widetilde{\xi})\mathbb{E}_1^i[p_1^U].$$

Averaging and iterating, with the same $\mathcal{G}_{1,\psi}$ as in (47), gives

$$p_1^U = \frac{\psi}{1-\psi}\left(\mathcal{G}_{1,\psi} - g^H\right).$$

Boundedness and $0 < 1 - \widetilde{\xi} < 1$ again imply uniqueness. Substituting this price index into the output equation yields

$$y_1^U = \psi\left(g_1 - \mathcal{G}_{1,\psi}\right).$$

The domestic-currency output gap in (47) is $g_1 - \mathcal{G}_{1,\psi}$. The policy set, prior, and signals are the same across arrangements, so the belief hierarchy defining $\mathcal{G}_{1,\psi}$ is also the same. Hence $y_{1,\psi}^{K,U} = \psi y_{1,\psi}^K$. The inequalities follow from $0 < \psi < 1$ and the signs in Proposition 7.

Part (b). Substituting $a_1 = \psi(g^H - g_1)$ into (41) gives

$$y_1 = -(1-\psi)p_1^U.$$

The optimality condition for a price chosen in URV becomes

$$x_1(i) = \mathbb{E}_1^i[p_1^U] + \xi\mathbb{E}_1^i[y_1] = [1 - \xi(1-\psi)]\,\mathbb{E}_1^i[p_1^U].$$

Iterating the aggregate relation leaves a bounded remainder multiplied by $[1-\xi(1-\psi)]^n$, which converges to zero. Hence $p_1^U = 0$ is the unique bounded solution, every firm chooses $x_1(i) = 0$, and $y_1 = 0$. Inflation is $\pi_1 = u_1 = g_1$. This argument holds for every prior,

including the endpoints.

The monetary authority's losses under the two regimes are

$$\mathcal{L}_1(g^L;\theta) = 0, \qquad \mathcal{L}_1(g^H;\theta) = (g^H)^2 > 0.$$

Every type strictly chooses g^L, independently of firms' prior. Consistency requires $\lambda_0^* = 1$, making full credibility the unique URV equilibrium. Under stabilization, $g_1 = 0$ and $a_1 = \psi g^H$, so $\pi_1 = y_1 = 0$. □

F.3 Proof of Proposition 9

Proposition 9.
With lagged indexation, both output gaps are strictly decreasing in ω. In particular, for $0 < \omega < 1$,

$$y^L_{1,\omega} < y^L_{1,0} < 0 < y^H_{1,\omega} < y^H_{1,0}.$$

At $\omega = 1$, $\pi^H_{1,1} = \pi^L_{1,1} = g^H$, $y^H_{1,1} = 0$, and $y^L_{1,1} = -g^H$. Under common knowledge of the realized regime, stabilization still produces positive inflation and a negative output gap whenever $\omega > 0$, while continued high money growth leaves output at its natural level and inflation at g^H.

Proof. Because contract status is independent of signals, average expectations among optimizing firms have the same distribution as in the baseline. Define

$$\chi_\omega \equiv \omega + (1-\omega)\xi, \qquad \delta_\omega \equiv \frac{\omega}{\chi_\omega}, \qquad D_\omega(\lambda_0) \equiv 1 - (1-\chi_\omega)\rho(\lambda_0),$$

where $\alpha(\lambda_0)$, $\beta(\lambda_0)$, and $\rho(\lambda_0)$ are the posterior moments defined in Appendix C.3. Here $\chi_\omega \in [\xi, 1]$, and δ_ω measures the contribution of inherited inflation after accounting for optimizing firms' responses. Let $y^K_{1,\omega}(\lambda_0)$ and $\pi^K_{1,\omega}(\lambda_0)$ denote the domestic-currency outcome under regime $K \in \{H, L\}$.

Averaging the pricing rules gives (44). Since $1 - \chi_\omega = (1-\omega)(1-\xi) \in [0, 1)$, iteration leaves a bounded remainder multiplied by $(1-\chi_\omega)^n$, which converges to zero. Define

$$\mathcal{G}_{1,\omega} \equiv \sum_{k=1}^{\infty} \chi_\omega (1-\chi_\omega)^{k-1} g_1^{(k)}.$$

The unique bounded aggregate solution is

$$\pi_1 = \delta_\omega g^H + (1-\delta_\omega)\mathcal{G}_{1,\omega}, \qquad y_1 = g_1 - \delta_\omega g^H - (1-\delta_\omega)\mathcal{G}_{1,\omega}. \tag{48}$$

Each indexed price is fixed by its contract, and the optimality condition uniquely determines every optimizing firm's price given this aggregate outcome.

Using (32) and summing the geometric series gives

$$\sum_{k=1}^{\infty} \chi_\omega (1-\chi_\omega)^{k-1} \bar{\lambda}_{1,k}^H = \frac{\alpha}{D_\omega},$$
$$\sum_{k=1}^{\infty} \chi_\omega (1-\chi_\omega)^{k-1} \bar{\lambda}_{1,k}^L = \frac{\alpha + \chi_\omega \rho}{D_\omega}.$$

Here and below, the argument λ_0 is suppressed. Since $g_{1,K}^{(k)} = g^H(1-\bar{\lambda}_{1,k}^K)$,

$$\mathcal{G}_{1,\omega}^H = g^H \left(1 - \frac{\alpha}{D_\omega}\right), \qquad \mathcal{G}_{1,\omega}^L = g^H \frac{1-\beta}{D_\omega}. \tag{49}$$

Substitution into (48) gives

$$y_{1,\omega}^H(\lambda_0) = (1-\delta_\omega) g^H \frac{\alpha(\lambda_0)}{D_\omega(\lambda_0)},$$
$$y_{1,\omega}^L(\lambda_0) = -g^H \left[\delta_\omega + (1-\delta_\omega)\frac{1-\beta(\lambda_0)}{D_\omega(\lambda_0)}\right], \qquad \pi_{1,\omega}^K(\lambda_0) = g^K - y_{1,\omega}^K(\lambda_0).$$

To establish monotonicity, collect the two state-contingent inflation rates in the vector $\boldsymbol{\pi}_\omega$, let $\boldsymbol{g} = (g^H, 0)'$, and write the average-expectations operator as

$$Q = \begin{pmatrix} 1-\alpha & \alpha \\ 1-\beta & \beta \end{pmatrix}.$$

The matrix Q is row stochastic and has strictly positive entries. The aggregate pricing equation becomes

$$\boldsymbol{\pi}_\omega = \omega g^H \mathbf{1} + (1-\omega)\left[\xi Q \boldsymbol{g} + (1-\xi) Q \boldsymbol{\pi}_\omega\right].$$

For $\omega < 1$, equation (48) and noisy informative signals imply that both components of $\boldsymbol{\pi}_\omega$ lie strictly between zero and g^H. Differentiating while holding the prior and signals fixed gives

$$\frac{\partial \boldsymbol{\pi}_\omega}{\partial \omega} = \left[I - (1-\chi_\omega)Q\right]^{-1} \frac{g^H \mathbf{1} - \boldsymbol{\pi}_\omega}{1-\omega}.$$

The inverse is the convergent series $\sum_{j=0}^{\infty}(1-\chi_\omega)^j K^j$, whose entries are nonnegative. The vector on its right is strictly positive, so both inflation rates are strictly increasing and both output gaps are strictly decreasing in ω. Continuity extends the comparisons to the extreme

cases. At $\omega = 0$, we have the baseline case, and at $\omega = 1$, contracts imply $\pi_1 = g^H$. The signs for $\omega < 1$ follow from the output formulas and $0 < \alpha < \beta < 1$.

Under common knowledge of the realized regime, $g_1^{(k)} = g^K$ at every order. The weights defining $\mathcal{G}_{1,\omega}$ sum to one, so $\mathcal{G}_{1,\omega} = g^K$. Substituting into (48) gives

$$\pi_{1,\omega}^K = \delta_\omega g^H + (1 - \delta_\omega) g^K, \qquad y_{1,\omega}^K = \delta_\omega (g^K - g^H).$$

For $\omega > 0$, $\delta_\omega > 0$, so stabilization produces positive inflation and a negative output gap even under common knowledge. Continued high money growth gives $\pi_{1,\omega}^H = g^H$ and $y_{1,\omega}^H = 0$. □

F.4 Proof of Proposition 10

Proposition 10.
Suppose optimizing firms set prices in URV with $u_1 = g_1$. Then,

(a) If inherited contracts retain their nominal lagged-indexation clause, $\lambda_0 \in (0, 1)$, and $0 < \omega < 1$, the unique equilibrium satisfies

$$y_{1,\omega}^L < y_{1,\omega}^{L,U} < 0, \qquad y_{1,\omega}^{H,U} < 0 < y_{1,\omega}^H.$$

Under common knowledge of the realized regime, the output gaps coincide with those under domestic-currency pricing.

(b) If inherited contracts are converted according to (46), then for any $0 \leq \omega \leq 1$ and any prior, the unique equilibrium satisfies $x_1(i) = p_1^U = y_1 = 0$ and $\pi_1 = g^K$. Every monetary authority type chooses g^L, and the unique equilibrium has $\lambda_0^* = 1$ and $\pi_1 = y_1 = 0$.

Proof. Since $u_1 = g_1$, the quantity equation gives $y_1 = -p_1^U$, and the pricing condition for an optimizing firm in URV is

$$x_1(i) = \mathbb{E}_1^i[p_1^U] + \xi \mathbb{E}_1^i[y_1] = (1 - \xi)\mathbb{E}_1^i[p_1^U].$$

Part (a). Indexed firms have $x_1(i) = g^H - g_1$. Aggregating gives (45). Since $1 - \chi_\omega \in [0, 1)$,

its unique bounded solution is

$$\begin{aligned} p_1^U &= \omega \sum_{k=0}^{\infty} (1-\chi_\omega)^k (g^H - g_1)^{(k)} \\ &= \delta_\omega \left[g^H - \chi_\omega g_1 - (1-\chi_\omega)\mathcal{G}_{1,\omega} \right], \end{aligned}$$

where the zeroth-order object is the realized variable and $\mathcal{G}_{1,\omega}$ is defined in the proof of Proposition 9. The optimizing firms' prices are then uniquely determined by their optimality conditions. Using (49) and $y_1^U = -p_1^U$ gives

$$\begin{aligned} y_{1,\omega}^{H,U} &= -\delta_\omega g^H \frac{(1-\chi_\omega)\alpha}{D_\omega}, \\ y_{1,\omega}^{L,U} &= -\delta_\omega g^H \left[1 - \frac{(1-\chi_\omega)(1-\beta)}{D_\omega} \right] = -\delta_\omega g^H \frac{\chi_\omega + (1-\chi_\omega)\alpha}{D_\omega}. \end{aligned}$$

Both gaps are strictly negative for $0 < \omega < 1$, since $\delta_\omega > 0$, $1 - \chi_\omega > 0$, and $\alpha > 0$.

Subtracting the domestic-currency output gap in (48) yields

$$y_{1,\omega}^{K,U} - y_{1,\omega}^{K} = (1-\omega)(\mathcal{G}_{1,\omega}^{K} - g^K).$$

Under stabilization, $\mathcal{G}_{1,\omega}^{L} > 0$, so the URV output gap is strictly higher than the domestic-currency gap. Under continued high growth, $\mathcal{G}_{1,\omega}^{H} < g^H$, so the difference is negative. Combining these comparisons with the signs above and Proposition 9 gives the stated inequalities.

Under common knowledge, $\mathcal{G}_{1,\omega} = g^K$, so

$$p_1^U = \delta_\omega (g^H - g^K), \qquad y_1^U = \delta_\omega (g^K - g^H).$$

The contractual output cost is therefore unchanged under common knowledge if the nominal indexation clauses remain binding.

Part (b). Converted contracts fix indexed firms' prices in URV at $x_1(i) = 0$. Averaging over both groups gives

$$p_1^U = (1-\omega)(1-\xi)(p_1^U)^{(1)}.$$

Since $(1-\omega)(1-\xi) < 1$, iteration and boundedness imply $p_1^U = 0$ for any prior, including the endpoints. The pricing condition and converted contracts then give $x_1(i) = 0$ for every firm, while $y_1 = 0$ and $\pi_1 = u_1 = g_1$.

The authority's losses are zero under g^L and $(g^H)^2 > 0$ under g^H. Every type strictly chooses stabilization for any prior, so consistency uniquely requires $\lambda_0^* = 1$. □

G Details for the Quantitative Framework

This appendix provides the details for the model in Section 5. It is a bridge from the equations in the main text to the computation of the model used in the disinflation experiments.

G.1 Household Optimality and Discounting

Let ϱ_t be the multiplier on the household's budget constraint. The first-order conditions for consumption, labor, and real balances are

$$C_t^{-\gamma} = \varrho_t, \qquad \alpha = \varrho_t(1-\tau_t)\frac{W_t}{P_t}, \qquad \chi h_t^{-1/\zeta} = \varrho_t R_t,$$

where R_t is the nominal interest rate, the return the household gives up by holding money. Combining them gives (16):

$$(1-\tau_t)W_t = \alpha P_t C_t^{\gamma}, \qquad h_t = \left(\frac{\chi C_t^{\gamma}}{R_t}\right)^{\zeta}.$$

Cost minimization across goods gives $C_t(i) = C_t\,(P_t(i)/P_t)^{-\eta}$. Replacing P_t with the labor-supply condition gives the demand function in Section 5.

The discount factor. The household values one unit of domestic currency at date t at $Q_t = e^{-rt}C_t^{-\gamma}/P_t$. By the labor-supply condition,

$$Q_t = \frac{\alpha\, e^{-rt}}{(1-\tau_t)W_t}. \tag{50}$$

The nominal discount factor is the inverse of the after-tax nominal wage, discounted at r. I use this result three times.

First, it gives the nominal interest rate. By (20), the after-tax nominal wage grows at rate g^K in regime K, so $dQ_t/Q_t = -(r+g^K)\,dt$. The rate on an instantaneous nominal claim satisfies $R_t\,dt = -\mathbb{E}_t[dQ_t/Q_t]$. If the regime is known, $R_t = R^K \equiv r + g^K$. If the household assigns probability $\bar{\lambda}_t$ to regime L, the expectation gives (21).

Second, it gives the real interest rate on indexed debt. Along the path of a regime, $r_t = r + \gamma\,\dot{C}_t/C_t$.

Third, it gives the firm's objective. Let $\Pi_t^n(i)$ be the nominal profit of firm i. The firm is owned by the household, so its nominal value is $\mathbb{E}_t\int_t^{\infty}(Q_s/Q_t)\,\Pi_s^n(i)\,ds$. Dividing by the

after-tax nominal wage and using (50),

$$\frac{1}{(1-\tau_t)W_t}\,\mathbb{E}_t\int_t^\infty \frac{Q_s}{Q_t}\,\Pi_s^n(i)\,ds = \mathbb{E}_t\int_t^\infty e^{-r(s-t)}\,\frac{\Pi_s^n(i)}{(1-\tau_s)W_s}\,ds. \tag{51}$$

Measured in units of the after-tax nominal wage, profits are discounted at the constant rate r. This is the unit in which the value function v_t and the profit function Π of Section 5.3 are expressed. It is why the firm discounts at r although the real interest rate varies, and why the tax rate is an argument of Π.

Money and inflation. By (16), the inverse of velocity is $k_t = \chi^\zeta C_t^{\gamma(\zeta-1)} R_t^{-\zeta}$. The money rule (19) then implies, on differentiable paths,

$$d\log M_t = g^K dt - \zeta\, d\log R_t + \gamma(\zeta-1)\, d\log C_t.$$

The money stock grows at g^K only when the nominal rate and consumption are constant. While the nominal rate falls, the monetary authority issues the money that households want to hold. This issuance is the remonetization term in (23). Since the rule implies $P_t C_t^\gamma = \bar{M}_t/k_{0^-}$, inflation is

$$\pi_t^K = g^K - \gamma\,\frac{\dot{C}_t^K}{C_t^K}.$$

G.2 The Profit Function

Let Y_t be aggregate output and $w_t \equiv W_t/P_t$ the real wage. Firm i sells $Y_t(i) = Y_t\,(P_t(i)/P_t)^{-\eta}$ and its nominal profit is $\Pi_t^n(i) = [P_t(i) - Z_t(i)W_t]\,Y_t(i)$. The definition of the markup, $x_t(i) = \log[P_t(i)/(Z_t(i)W_t)]$, gives $P_t(i)/W_t = Z_t(i)e^{x_t(i)}$. Hence

$$\begin{aligned}\Pi_t^n(i) &= W_t Y_t w_t^{-\eta}\left(\frac{P_t(i)}{W_t}\right)^{-\eta}\left(\frac{P_t(i)}{W_t} - Z_t(i)\right)\\ &= W_t Y_t w_t^{-\eta}\, Z_t(i)^{1-\eta}\left[e^{(1-\eta)x_t(i)} - e^{-\eta x_t(i)}\right].\end{aligned}$$

The term in brackets is maximized at the frictionless markup $x^f = \log[\eta/(\eta-1)]$, where it equals $\kappa_\eta \equiv (\eta-1)^{\eta-1}/\eta^\eta$. Define

$$\Lambda(x) \equiv \frac{1}{\kappa_\eta}\left[e^{(1-\eta)x} - e^{-\eta x}\right],$$

so that $\Lambda(x^f) = 1$. $\Lambda(x)$ is the share of the frictionless profit that a firm with markup x earns. Dividing nominal profits by the after-tax nominal wage, as in (51),

$$\frac{\Pi_t^n(i)}{(1-\tau_t)W_t} = A_t\, e^{(1-\eta)z_t(i)}\, \Lambda\big(x_t(i)\big), \qquad A_t \equiv \kappa_\eta\, \frac{Y_t w_t^{-\eta}}{1-\tau_t}. \tag{52}$$

Profits are the product of three terms. The aggregate term A_t depends on output, the real wage, and the tax rate. The term $e^{(1-\eta)z}$ is the scale of the firm: a firm with a lower cost sells more. The term $\Lambda(x)$ depends only on the markup.

The profit function of Section 5.3 is (52) for a firm of unit scale:

$$\Pi(x; Y_t, w_t, \tau_t) = A_t\, \Lambda(x), \qquad \bar{\Pi}(Y_t, w_t, \tau_t) = A_t.$$

I show in Appendix G.5 that the scale factors out of the firm's problem. In units of the consumption good, the frictionless profit of a firm with cost z is

$$\phi_t(z) = \kappa_\eta Y_t w_t^{1-\eta} e^{(1-\eta)z}, \tag{53}$$

and the menu cost is $c\,\phi_t(z)$. In units of the after-tax nominal wage, the menu cost is $c\, A_t e^{(1-\eta)z}$.

G.3 Markup Dynamics and the Announcement Date

Between price changes, the markup of a firm moves for three reasons: its own cost shock, the change in the nominal wage, and, when its price is posted in a unit of account, the change in the conversion factor. Section 5.6 gives the law of motion under each instrument. In regime K, they are all of the form

$$dx_t(i) = a_t^{j,K}\, dt + \beta^j\, dB_t^\varepsilon - \sigma_z\, dB_t^z(i), \tag{54}$$

with

$$a_t^{j,K} = \begin{cases} -g^K + \dfrac{d\log(1-\tau_t^K)}{dt}, & j = N, \\ -\dfrac{d\log w_t^K}{dt}, & j = U, \\ -g^L + \dfrac{d\log(1-\tau_t^L)}{dt}, & j = E, \end{cases} \qquad \beta^j = \begin{cases} 0, & j \in \{N, U\}, \\ \sigma_\varepsilon, & j = E. \end{cases}$$

Under nominal pricing the posted price is fixed in domestic currency, so the markup falls with the nominal wage, which grows according to (20). Under the URV the posted price is

a relative price, $X_t^U(i) = P_t(i)/P_t$, so the markup is $x_t^U(i) = \log X_t^U(i) - z_t(i) - \log w_t$ and it moves only with the real wage. Money growth does not enter. Under the peg the markup falls with the nominal wage, as under nominal pricing, and it also moves with the exchange rate. Since the peg is adopted only in regime L, only $K = L$ is relevant for $j = E$.

The announcement date. Before $t = 0$ the regime is known to be H. At $t = 0$, beliefs move to the prior λ_0 under N and U, and to one under E. By (21) the nominal rate falls, so k_t jumps.

Under the URV and the peg, the money rule (19) applies at $t = 0$. The money stock jumps with money demand, and the monetary authority sets $\bar{M}_0 = \bar{M}_{0^-}(1-\tau_0)/(1-\tau_{0^-})$, so that the nominal wage is continuous, $W_0 = W_{0^-}$. Posted prices are converted at U_0^j at no cost and relative prices are preserved. No markup moves at the announcement.

In the money-based stabilization of Section 5.9, the money stock is continuous at $t = 0$, $M_0 = M_{0^-}$, and the rule applies from then on. Since $(1-\tau_t)W_t = \alpha M_t/k_t$, the nominal wage falls at the announcement by the increase in money demand:

$$\Delta \log W_0 = -\Delta \log k_0 - \Delta \log(1-\tau_0) = \log \frac{R_0}{R^H} - \log \frac{1-\tau_0}{1-\tau_{0^-}},$$

where the second equality uses $\zeta = 1$. Prices are fixed in domestic currency, so every markup rises by the same amount, $x_0(i) = x_{0^-}(i) - \Delta \log W_0$.

G.4 Beliefs

This subsection derives (28) and the law of motion of beliefs that enters the firm's problem. I suppress the instrument superscript. For $a \in \{g, \tau\}$, the signals of Section 5.7 are

$$dS_t^a(i) = \mu_{a,t}^K \, dt + \sigma_a \, dB_t^a(i), \qquad \mu_{g,t}^K = g^K, \qquad \mu_{\tau,t}^K = \tau_t^K. \tag{55}$$

Conditional on regime K, each increment is normal with mean $\mu_{a,t}^K dt$ and variance $\sigma_a^2 dt$, and the two increments are independent. The log of the likelihood ratio of regime L to regime H is therefore

$$\begin{aligned}
\log \frac{f_L(dS_t(i))}{f_H(dS_t(i))} &= \sum_{a\in\{g,\tau\}} \frac{\left(dS_t^a(i) - \mu_{a,t}^H dt\right)^2 - \left(dS_t^a(i) - \mu_{a,t}^L dt\right)^2}{2\sigma_a^2 dt} \\
&= \sum_{a\in\{g,\tau\}} \frac{\mu_{a,t}^L - \mu_{a,t}^H}{\sigma_a^2} \left[dS_t^a(i) - \frac{\mu_{a,t}^L + \mu_{a,t}^H}{2} \, dt \right].
\end{aligned}$$

By Bayes' rule, the posterior log-odds $\ell_t(i)$ increase by the log-likelihood ratio, which gives (28):

$$d\ell_t(i) = \sum_{a\in\{g,\tau\}} \frac{\mu_{a,t}^L - \mu_{a,t}^H}{\sigma_a^2} \left[dS_t^a(i) - \frac{\mu_{a,t}^L + \mu_{a,t}^H}{2}\, dt \right]. \tag{56}$$

Define the information rate

$$I_t \equiv \frac{(g^L - g^H)^2}{\sigma_g^2} + \frac{(\tau_t^L - \tau_t^H)^2}{\sigma_\tau^2}. \tag{57}$$

Substituting (55) into (56), the log-odds in regime K follow

$$d\ell_t(i) = \mu_{\ell,t}^K\, dt + \sqrt{I_t}\, dB_t^\ell(i), \qquad \mu_{\ell,t}^L = \frac{I_t}{2}, \qquad \mu_{\ell,t}^H = -\frac{I_t}{2}, \tag{58}$$

where $B_t^\ell(i)$ is a Brownian motion that combines the two signal innovations. Beliefs drift toward the truth at half the information rate. The first term of I_t is constant. The second term depends on how different the tax paths of the two regimes are, so it is determined in equilibrium.

The firm does not know the regime, so it does not know the drift of its own belief. With posterior $\lambda(\ell)$, it expects the drift

$$\widehat{\mu}_{\ell,t}(\ell) = \lambda(\ell)\frac{I_t}{2} - \big(1 - \lambda(\ell)\big)\frac{I_t}{2} = \big(2\lambda(\ell) - 1\big)\frac{I_t}{2}. \tag{59}$$

With this drift, Itô's lemma applied to $\lambda(\ell)$ gives a posterior probability with zero drift: the firm does not expect to change its mind. Under the credible peg, $\lambda = 1$ and there is no learning.

G.5 The Firm's Problem

This subsection writes (18) explicitly. The computations add one feature to the model of Section 5. Each product is replaced at rate $\rho \geq 0$ by a new product, which keeps the cost level and the belief of the firm and whose price is set at the reset markup without paying the menu cost. The firm values a product until it is replaced. Setting $\rho = 0$ gives the model in the main text.[17]

The problem with the cost level as a state. By (52), the profit of a firm depends on its markup x, its cost z, and the aggregate term A_t. The firm knows the path A_t^K in each

[17] In a stationary equilibrium, product replacement gives the same pricing policy as an opportunity to change the price at no cost that arrives at rate ρ. The two differ by a constant in the value function.

regime but not the regime, so with log-odds ℓ it expects

$$\widehat{A}_t(\ell) = \lambda(\ell)A_t^L + \big(1-\lambda(\ell)\big)A_t^H, \qquad \widehat{a}_t^j(\ell) = \lambda(\ell)a_t^{j,L} + \big(1-\lambda(\ell)\big)a_t^{j,H},$$

where $a_t^{j,K}$ is the drift of the markup in (54). Let $V_t(x,z,\ell)$ be the value of the firm in units of the after-tax nominal wage. It solves

$$\begin{aligned}\max\Big\{ &\widehat{A}_t(\ell)e^{(1-\eta)z}\Lambda(x) + \partial_t V_t + \mathcal{L}_t V_t - (r+\rho)V_t, \\ &\max_{x'} V_t(x',z,\ell) - c\,\widehat{A}_t(\ell)e^{(1-\eta)z} - V_t\Big\} = 0,\end{aligned} \tag{60}$$

where the generator of (x,z,ℓ) is, from (54), $dz_t = \sigma_z dB_t^z$, and (59),

$$\mathcal{L}_t V = \widehat{a}_t^j(\ell)\,\partial_x V + \frac{\sigma_z^2 + (\beta^j)^2}{2}\,\partial_{xx}V + \frac{\sigma_z^2}{2}\,\partial_{zz}V - \sigma_z^2\,\partial_{xz}V + \widehat{\mu}_{\ell,t}(\ell)\,\partial_\ell V + \frac{I_t}{2}\,\partial_{\ell\ell}V.$$

The cross term appears because a cost shock raises z and lowers x by the same amount.

The cost level factors out. Profits and the menu cost are proportional to $e^{(1-\eta)z}$, and the law of motion of z does not depend on its level. Hence the value function is proportional to $e^{(1-\eta)z}$. Guess $V_t(x,z,\ell) = e^{(1-\eta)z}\,u_t(x,\ell)$. Then $\partial_z V = (1-\eta)V$, $\partial_{zz}V = (1-\eta)^2 V$, and $\partial_{xz}V = (1-\eta)e^{(1-\eta)z}\partial_x u$, so

$$\frac{\sigma_z^2}{2}\,\partial_{zz}V - \sigma_z^2\,\partial_{xz}V = e^{(1-\eta)z}\left[\vartheta\,u + (\eta-1)\sigma_z^2\,\partial_x u\right], \qquad \vartheta \equiv \frac{(\eta-1)^2\sigma_z^2}{2}.$$

Removing the cost level from the state leaves two terms. The first is a growth term, ϑu: the expected scale of the firm, $\mathbb{E}_t[e^{(1-\eta)z_s}]$, grows at rate ϑ. The second is an additional drift of the markup, $(\eta-1)\sigma_z^2$. The firm earns more in the states in which its cost has fallen, and in those states its markup has risen. A firm that weights states by its profits therefore expects its markup to rise.

The trend. The growth term is offset by a trend in the aggregate term. Since $z_t(i)$ is a random walk, $\int_0^1 Z_t(i)^{1-\eta}di = e^{\vartheta t}$, where I normalize its initial value to one. The dispersion of costs grows over time, and with it the productivity of the economy, because households buy more from the firms with lower costs. With $\gamma = \zeta = 1$, the values I use, the real wage, consumption, output, and real balances grow at rate $\vartheta/(\eta-1)$, which is 1.4% a year at the parameters of Table 2. Labor is constant, and A_t falls at rate ϑ. I measure every real quantity relative to this trend and keep the notation of Section 5. The equations of that section hold for the detrended variables, with inflation measured net of the trend, and the

stationary equilibria of the text are stationary in these variables. In the firm's problem, the decline of A_t at rate ϑ cancels the growth term. Formally, $V_t(x,z,\ell) = e^{(1-\eta)z-\vartheta t}\, v_t(x,\ell)$, where v_t is defined with the detrended A_t.

The equation that is solved. Substituting into (60), $v_t(x,\ell)$ solves

$$\begin{aligned}\max\Bigg\{ &\widehat{A}_t(\ell)\Lambda(x) + \partial_t v_t + \left[\widehat{a}_t^j(\ell) + (\eta-1)\sigma_z^2\right]\partial_x v_t + \frac{\sigma_z^2 + (\beta^j)^2}{2}\,\partial_{xx} v_t \\ &+ \mathcal{B}_t v_t - (r+\rho)v_t, \quad \max_{x'} v_t(x',\ell) - c\,\widehat{A}_t(\ell) - v_t \Bigg\} = 0,\end{aligned} \tag{61}$$

with

$$\mathcal{B}_t v = \big(2\lambda(\ell) - 1\big)\frac{I_t}{2}\,\partial_\ell v + \frac{I_t}{2}\,\partial_{\ell\ell} v. \tag{62}$$

This is (18), with the belief written in log-odds. Expected profits are $\widehat{\Pi}_t(x,\lambda) = \widehat{A}_t(\ell)\Lambda(x)$, the expected frictionless profit is $\widehat{\bar{\Pi}}_t(\lambda) = \widehat{A}_t(\ell)$, and the expected erosion of the markup is the drift $\widehat{a}_t^j(\ell)$. Under nominal pricing this drift is minus the expected growth of the nominal wage, $-[\lambda g^L + (1-\lambda)g^H]$ plus the expected change in $\log(1-\tau_t)$. The term $\mathcal{B}_t v_t$ is (62). Equation (61) has two terms that (18) does not display: the drift $(\eta-1)\sigma_z^2$ from the cost level, and the replacement rate ρ.

The value function depends on time because $\widehat{A}_t$, $\widehat{a}_t^j$, and I_t do. The firm resets to $x_t^*(\ell) = \operatorname{argmax}_{x'} v_t(x',\ell)$ when $v_t(x,\ell) = v_t(x_t^*(\ell),\ell) - c\,\widehat{A}_t(\ell)$. The reset markup and the inaction region do not depend on the cost level. A firm that resets posts the price $\log P_t(i) = x_t^*(\ell) + z_t(i) + \log W_t$, or $\log X_t^j(i) = \log P_t(i) - \log U_t^j$ in the unit of account.

The three instruments. Under nominal pricing and the URV the firm solves (61) with $\beta^j = 0$. The two differ in the drift. Under nominal pricing the firm's belief about the regime moves its expected markup drift one for one with expected money growth. Under the URV the drift is minus the expected change in the real wage, and money growth affects the firm only through the paths of the real wage and of A_t. Under the credible peg the belief is fixed at $\lambda = 1$, the term $\mathcal{B}_t v_t$ drops, and the firm solves a problem in x alone with A_t^L, $a_t^{E,L}$, and variance $\sigma_z^2 + \sigma_\varepsilon^2$. The exchange rate enters the firm's problem only through this larger variance: given the path of A_t, the level of the exchange rate is not a state, because the firm observes its markup.

G.6 The Distribution of Firms

Aggregate variables are integrals over firms of $Z_t(i)^{1-\eta}$ times a function of the markup (Appendix G.7). I therefore do not need the joint distribution of markups, costs, and beliefs, but only the distribution of markups and beliefs with each firm weighted by its scale. Let $f_t^K(x, z, \ell)$ be the density of firms in regime K and define

$$m_t^K(x, \ell) \equiv e^{-\vartheta t} \int e^{(1-\eta)z} f_t^K(x, z, \ell)\, dz,$$

which integrates to one. Multiplying the law of motion of f_t^K by $e^{(1-\eta)z}$ and integrating over z by parts gives the law of motion of m_t^K. Inside the inaction region,

$$\begin{aligned} \partial_t m_t^K = &- \partial_x \Big(\big[a_t^{j,K} + (\eta - 1)\sigma_z^2 \big] m_t^K \Big) + \frac{\sigma_z^2}{2}\, \partial_{xx} m_t^K \\ &- \partial_\ell \big(\mu_{\ell,t}^K m_t^K \big) + \frac{I_t}{2}\, \partial_{\ell\ell} m_t^K - \rho\, m_t^K. \end{aligned} \tag{63}$$

The weighting adds the drift $(\eta - 1)\sigma_z^2$, the same that appears in the firm's problem. The density evolves with the true drifts of regime K, $a_t^{j,K}$ and $\mu_{\ell,t}^K$ from (58), while firms choose their policies with the expected drifts. Firms that reach the boundary of the inaction region, and the products that are replaced, are reinjected at the reset markup $x_t^*(\ell)$ with the same belief. Let $\mathcal{J}_t^K$ be the weighted mass of firms that reach the boundary per unit of time. These are the price changes that pay the menu cost.

At $t = 0$ all firms have the prior, and the distribution of markups is the stationary one of regime H, shifted by $-\Delta \log W_0$ in the money-based stabilization (Appendix G.3).

Signals are independent of costs, so the weighting does not change the distribution of beliefs. The average belief in (29) is

$$\bar{\lambda}_t^K = \iint \lambda(\ell)\, m_t^K(x, \ell)\, dx\, d\ell.$$

The peg. The exchange-rate shock is common to all firms, so under the peg the distribution of markups, and with it every aggregate variable, depends on the realized path of the exchange rate. I report the path along which the realized innovations are zero. On this path the density follows (63) with $K = L$, without the belief terms, and with variance σ_z^2. Firms take exchange-rate risk into account when they set prices, through the variance $\sigma_z^2 + \sigma_\varepsilon^2$ in (61), and they take the path of A_t on the reported path as given. The peg therefore differs from a regime-L economy with nominal pricing and a known regime only through wider inaction regions.

G.7 Aggregation and Market Clearing

Three moments of m_t^K summarize the pricing decisions of firms. The first comes from the price index. Since $P_t(i)/W_t = Z_t(i)e^{x_t(i)}$,

$$\left(\frac{P_t}{W_t}\right)^{1-\eta} = \int_0^1 Z_t(i)^{1-\eta} e^{(1-\eta)x_t(i)}\, di = e^{\vartheta t}\,\Omega_t^K, \qquad \Omega_t^K \equiv \iint e^{(1-\eta)x}\, m_t^K(x,\ell)\, dx\, d\ell.$$

The detrended real wage is

$$w_t^K = \left(\Omega_t^K\right)^{\frac{1}{\eta-1}}. \tag{64}$$

The real wage is low when markups are high. The second moment comes from labor demand. Firm i uses $L_t(i) = Z_t(i)Y_t(i) = Y_t w_t^{-\eta} Z_t(i)^{1-\eta} e^{-\eta x_t(i)}$ units of labor. Integrating over firms, labor-market clearing requires

$$L_t^K = Y_t^K \left(w_t^K\right)^{-\eta} \mathcal{M}_t^K, \qquad \mathcal{M}_t^K \equiv \iint e^{-\eta x}\, m_t^K(x,\ell)\, dx\, d\ell. \tag{65}$$

The dispersion of markups raises $\mathcal{M}_t^K$ relative to Ω_t^K, and with it the labor needed to produce one unit of output. This is the misallocation cost of price dispersion. The third moment is the rate of costly price changes, $\mathcal{J}_t^K$. Each one costs $c\,\phi_t(z)$ units of the consumption good, with ϕ_t from (53), so total menu costs are

$$\Gamma_t^K = c\,\kappa_\eta Y_t^K \left(w_t^K\right)^{1-\eta} \mathcal{J}_t^K \equiv \theta_t^K Y_t^K.$$

Goods-market clearing requires

$$Y_t^K = C_t^K + G_t + \Gamma_t^K, \tag{66}$$

so $Y_t^K = (C_t^K + G_t)/(1-\theta_t^K)$.

Given $(w_t^K, \mathcal{M}_t^K, \theta_t^K)$, the nominal rate R_t^K from (21) at the average belief $\bar{\lambda}_t^K$, and a tax rate τ, the allocation follows in closed form:

$$\begin{aligned} C &= \left(\frac{(1-\tau)w_t^K}{\alpha}\right)^{1/\gamma}, \qquad Y = \frac{C+G_t}{1-\theta_t^K}, \qquad L = Y\left(w_t^K\right)^{-\eta}\mathcal{M}_t^K, \\ h &= \left(\frac{\chi C^\gamma}{R_t^K}\right)^{\zeta}, \qquad T = \tau\, w_t^K L, \qquad A = \kappa_\eta \frac{Y\left(w_t^K\right)^{-\eta}}{1-\tau}. \end{aligned} \tag{67}$$

The tax lowers consumption and output at given prices, because it lowers labor supply. I write $C_t^K(\tau)$, $L_t^K(\tau)$, $h_t^K(\tau)$, and $T_t^K(\tau)$ for these functions.

G.8 The Government's Problem

The government knows the regime. In each regime it takes as given the pricing moments $(w_t^K, \mathcal{M}_t^K, \theta_t^K)$ and the path of the nominal rate, and chooses the tax. It does not internalize the effect of the tax on firms' prices or on beliefs.

The budget. Add real balances to both sides of (22) and use (23):

$$d(b_t + h_t) = (r_t b_t + G_t - T_t - \pi_t h_t)\, dt.$$

The sum of debt and real balances does not jump. When real balances jump, the government retires debt by the same amount. With $r_t = r + \gamma \dot{C}_t/C_t$ and $\pi_t = g^K - \gamma \dot{C}_t/C_t$, the right-hand side is $rb_t - g^K h_t + G_t - T_t$ plus the term $\gamma(\dot{C}_t/C_t)(b_t + h_t)$, which I neglect in the computations. Then

$$d(b_t + h_t) = \left[r(b_t + h_t) - \left(T_t + R^K h_t - G_t\right)\right] dt,$$

and integrating from 0^- to $\bar{t}$ with the debt target (24) gives

$$b_0 + h_{0^-} = \int_0^{\bar{t}} e^{-rt} \left[T_t + R^K h_t - G_t\right] dt + e^{-r\bar{t}} \left(B + h_{\bar{t}}\right). \tag{68}$$

This is (25) with the remonetization term integrated by parts. Each unit of real balances that households hold is worth $R^K = r + g^K$ per unit of time to the government: the interest on the debt it replaces, plus seigniorage. The path of h_t determines the timing of remonetization. As real money demand goes up, remonetization revenue increases. The later households rebuild their real balances, the smaller the integral, and the more must come from taxes. The budget constraint (68) is the same for the three instruments.

The tax before $\bar{t}$. The government maximizes (26) subject to (68). Let $\mathcal{U}_t^K(\tau)$ be the flow utility of the household at the allocation (67), and let μ^K be the multiplier on the budget. The objective and the budget are discounted at the same rate, so the problem separates across dates:

$$\tau_t^K = \operatorname*{argmax}_{0 \le \tau \le \bar{\tau}} \left\{ \mathcal{U}_t^K(\tau) + \mu^K \left[T_t^K(\tau) + R^K h_t^K(\tau)\right] \right\}. \tag{69}$$

At an interior optimum, $-\partial_\tau \mathcal{U}_t^K = \mu^K \, \partial_\tau [T_t^K + R^K h_t^K]$. The welfare cost of one more unit of revenue is μ^K at every date, which is the tax-smoothing condition of Section 5.5. The tax still varies over time, because the pricing moments and the nominal rate do. The multiplier

is the value at which (68) holds.

The tax from $\bar{t}$ on. From $\bar{t}$ on, the economy is in the stationary equilibrium of its regime (Appendix G.9), real balances are constant, and (27) reduces to

$$T_{\bar{t}}^{K}(\tau) + g^{K} h_{\bar{t}}^{K}(\tau) - G = rB.$$

The tax is the lower of the two solutions, on the increasing side of the Laffer curve. The same condition with b_0 in place of B and $K = H$ gives the tax in the initial stationary equilibrium, τ_{0^-}.

The two regimes. The government faces the same target $(B, \bar{t})$ in both regimes. The tax paths differ because seigniorage and the path of real balances differ. In regime L the government loses seigniorage and collects the remonetization revenue. In regime H it keeps seigniorage, and real balances rise at the announcement and then return to their initial level as households learn that the regime has not changed. The gap between τ_t^L and τ_t^H is what makes the tax signal informative in (57).

In the exercise without a government, there is no debt, $\tau_t = 0$, and the revenue from money creation is rebated to the household as a lump sum.

G.9 Stationary Equilibria and Computation

Stationary equilibria. When the regime is known and the tax is constant, A_t is constant and $v = A\,\bar{v}^K(x)$, where $\bar{v}^K$ solves

$$\max\left\{\Lambda(x) + \left[-g^K + (\eta-1)\sigma_z^2\right]\partial_x \bar{v}^K + \frac{\sigma_z^2}{2}\,\partial_{xx}\bar{v}^K - (r+\rho)\bar{v}^K,\ \max_{x'}\bar{v}^K(x') - c - \bar{v}^K\right\} = 0.$$

The pricing policy does not depend on the level of output or on the level of the tax. The stationary density solves (63) with $\partial_t m = 0$ and without the belief terms, and the tax follows from the stationary budget. Under the URV the drift is $(\eta-1)\sigma_z^2$ in both regimes, because the real wage is constant. Under the peg the firm's variance is $\sigma_z^2 + \sigma_\varepsilon^2$.

The economy starts in the stationary equilibrium of regime H with debt b_0. From $\bar{t}$ on, it is in the stationary equilibrium of its regime with debt B, under the instrument in place. The terminal condition of (61) weights the two stationary values by the firm's belief at $\bar{t}$:

$$v_{\bar{t}}(x,\ell) = \lambda(\ell) A_{\bar{t}}^{L}\,\bar{v}^{L}(x) + \big(1-\lambda(\ell)\big) A_{\bar{t}}^{H}\,\bar{v}^{H}(x).$$

Algorithm. An equilibrium is a fixed point in the paths of the pricing moments and of the average belief in the two regimes. Starting from a guess of $\{w_t^K, \mathcal{M}_t^K, \theta_t^K, \bar{\lambda}_t^K\}$ for $K \in \{L, H\}$ and $t \in [0, \bar{t}]$:

1. Compute the nominal rate (21) in each regime from $\bar{\lambda}_t^K$. Solve the government's problem in each regime, (68) and (69). This gives τ_t^K and the allocation (67), including A_t^K.

2. Compute the drifts $a_t^{j,K}$ and the information rate I_t from the tax paths and, under the URV, from the real-wage paths.

3. Solve (61) backward from $\bar{t}$. This gives the reset markup $x_t^*(\ell)$ and the inaction region at each date.

4. Solve (63) forward from $t = 0$ in each regime, with the policies of step 3 and the true drifts of that regime. Compute Ω_t^K, $\mathcal{M}_t^K$, $\mathcal{J}_t^K$, and $\bar{\lambda}_t^K$.

5. Update the guess toward the new paths and repeat until the aggregate moments converge.

G.10 Parameters

Table 2 lists the parameters used in the disinflation experiments of Section 5.9. Time is measured in years. The volatility of cost shocks and the menu cost target, at zero inflation and without product replacement, about 2.7 price changes a year and an average price change of about 10%. The two economies with a government differ in the weight of money in utility, and therefore in how much revenue the government collects from money creation before the reform. In both, the debt target B is the level of debt once real balances have been fully rebuilt. The exercise without a government uses $\chi = 0.011$.

Table 2: Parameters of the quantitative model

Parameter	**Symbol**	**Value**
Preferences and technology		
Discount rate	r	0.06
Inverse of IES	γ	1
Disutility of labor	α	1
Elasticity of substitution	η	2
Interest elasticity of money demand	ζ	1
Weight of money in utility	χ	0.011
Price setting		
Volatility of cost shocks	σ_z	0.165
Menu cost, share of annual frictionless profits	c	0.0006
Rate of product replacement	ρ	0.8
Monetary policy and instruments		
Money growth in regime H	g^H	2
Money growth in regime L	g^L	0
Volatility of the exchange rate under the peg	σ_ε	0.10
Information		
Prior probability of regime L	λ_0	0.4
Noise in the monetary signal	σ_g	0.3
Noise in the tax signal	σ_τ	0.2
Fiscal policy		
Initial debt	b_0	0.4
Debt target	B	0.318 / 0.167
Date of the debt target	$\bar{t}$	5
Government purchases	G	0